



Fuzzy measures for profit maximization with fuzzy parameters

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ABSTRACT

Profit maximization is an important issue to the firms that pursue the largest economic profit possible. This paper extends the situation from the deterministic to uncertain, where the coefficients are represented by fuzzy numbers. Intuitively, when the problem has fuzzy parameters, the derived profit value should be a fuzzy number as well. The extension principle is utilized to develop a pair of two-level mathematical programs to calculate the upper and lower bounds of the profit value at α -cuts. Following the duality theorem and a variable separation technique, the two-level mathematical programs are transformed into a class of one-level signomial geometric programs to solve. An example is given to illustrate the idea proposed in this paper.

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1. Introduction

Economic profit is the difference between revenue from selling output and the cost of acquiring the factors necessary to produce it. A profit-maximizing firm chooses both its inputs and outputs with the sole goal of achieving maximum economic profits. That is, the firm seeks to maximize the difference between its total revenue and its total economic costs. If firms are strict profit maximizers, they will adjust those variables that can be controlled until it is impossible to increase profits further. Most production functions in the profit-maximization problem are represented as power functions. Traditionally, the profit maximization problem is solved by the classical method of calculus.

Geometric programming is a methodology for solving algebraic nonlinear optimization problems. One of the remarkable properties of geometric programming is that a problem with highly nonlinear constraints can be stated equivalently as one with only linear constraints. This is because there is a strong duality theorem for geometric programming problems. If the primal problem is in posynomial form, then a global minimizing solution to that problem can be obtained by solving the dual maximization. The dual constraints are linear, and linearly constrained programs are generally easier to solve than ones with nonlinear constraints. Its attractive structural properties as well as its elegant theoretical basis have led to a number of interesting applications and the development of numerous useful results.

When the production function is represented as a power function, the profit maximization problem can be treated as a geometric program. Liu [1,2] employs geometric programming to solve the profit-maximization problems. No differentiation is required. However, Liu's studies use deterministic models, i.e., the coefficients in the problems are crisp values. There are situations when the data cannot be collected without error. If some parameters are imprecise or uncertain, then some crisp values are usually assigned to those uncertain parameters to make the related solution procedures workable. Furthermore, traditional calculus methods are difficult to apply to solving the profit-maximization with imprecise coefficients. To deal quantitatively with imprecise information in making decisions, Zadeh [3] introduced the notion of fuzziness.

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Fuzzy set theory has been extensively employed in linear and nonlinear optimization problems [4–10]. Intuitively, when the coefficients in the profit-maximization problem are fuzzy numbers, the derived profit value should be a fuzzy number as well. Unlike previous researches, this paper employs geometric programming to find the long-run maximum profit for the production function with Cobb–Douglas form under consideration of fuzzy coefficients and quantity discount. Based on Zadeh's extension principle [11,3,12], we construct a pair of two-level mathematical programming models, based on which the upper bound and lower bound of the profit value are calculated at possibility level α . In other words, a fuzzy profit value for the objective function of a profit-maximization problem with fuzzy parameters is derived.

The rest of this paper is organized as follows. We first present the long-run profit maximization problem with fuzzy coefficients and quantity discount, and we formulate the problem in the form of two-level mathematical programs for calculating the bounds of the profit value at α -cuts. We then transform the two-level mathematical programs into the conventional one-level signomial geometric programs to solve. Finally, some conclusions are drawn from the discussion.

2. Basic concepts on fuzzy sets

Fuzzy sets are generalizations of crisp sets as a way of representing imprecise or vagueness in real world. A fuzzy set is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and otherwise fuzzy. Each fuzzy set is specified by a membership function, which assigns to each element in the universe of discourse a value within the unit interval $[0, 1]$. The assigned value called the degree or grade of membership, which specifies the extent to which a given element belongs to the fuzzy set or is related to a concept. If the assigned value is 0, then the given element does not belong to the set. If the assigned value is 1, then the element totally belongs to the set. If the values lies within the interval $(0, 1)$, then the element only partially belongs to the set. Therefore, any fuzzy set can be uniquely determined by its membership function.

Let X be the universe of discourse. A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be convex if and only if for all x_1 and x_2 in X there always exist [11,3,12]:

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)).$$

Fuzzy sets can also be represented by intervals, which are called α -level sets or α -cuts. The α -level sets A_α of a fuzzy set $\tilde{A}$ are defined as

$$\begin{aligned} (A)_\alpha &= [(A)_\alpha^L, (A)_\alpha^U] \\ &= \left[\min_x \{x, \mu_{\tilde{A}}(x) | \mu_{\tilde{A}}(x) \geq \alpha\}, \max_x \{x, \mu_{\tilde{A}}(x) | \mu_{\tilde{A}}(x) \geq \alpha\} \right]. \end{aligned}$$

According to Zadeh's extension principle [11,3,12], the fuzzy set $\tilde{A}$ can be expressed as

$$\tilde{A} = \bigcup_{\alpha} A_\alpha, \quad 0 < \alpha \leq 1.$$

Fuzzy numbers are special cases of fuzzy sets that are both convex and normal. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. Its membership function is piecewise continuous and satisfies the following conditions:

- (a) $\mu_{\tilde{A}}(x) = 0$ for each $x \notin [a, d]$,
- (b) $\mu_{\tilde{A}}(x)$ is non-decreasing on $[a, b]$ and non-increasing on $[c, d]$,
- (c) $\mu_{\tilde{A}}(x) = 1$ for each $x \in [b, c]$,

where $a \leq b \leq c \leq d$ are real numbers.

The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, which are often denoted as $\tilde{A} = (a, b, d)$ and $\tilde{B} = (a, b, c, d)$. Obviously, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers with $b = c$. The membership functions of these two types of fuzzy numbers are respectively defined as

$$\begin{aligned} \mu_{\tilde{A}}(x) &= \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ (d-x)/(d-b), & b \leq x \leq d, \\ 0, & \text{otherwise} \end{cases} \\ \mu_{\tilde{B}}(x) &= \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ (d-x)/(d-c), & c \leq x \leq d, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Their α -levels sets are $(A)_\alpha = [(A)_\alpha^L, (A)_\alpha^U]$ and $(B)_\alpha = [(B)_\alpha^L, (B)_\alpha^U]$. With Zadeh's extension principle [11,3,12], the addition, subtraction, multiplication and division of fuzzy numbers can be equivalently represented as follows [13,12]:

Addition: $(A)_\alpha (+) (B)_\alpha = [(A)_\alpha^L, (A)_\alpha^U] + [(B)_\alpha^L, (B)_\alpha^U] = [(A)_\alpha^L + (B)_\alpha^L, (A)_\alpha^U + (B)_\alpha^U]$.

Subtraction: $(A)_\alpha (-) (B)_\alpha = [(A)_\alpha^L, (A)_\alpha^U] - [(B)_\alpha^L, (B)_\alpha^U] = [(A)_\alpha^L - (B)_\alpha^U, (A)_\alpha^U - (B)_\alpha^L]$.

Multiplication: $(A)_\alpha (\bullet) (B)_\alpha = [(A)_\alpha^L, (A)_\alpha^U] (\bullet) [(B)_\alpha^L, (B)_\alpha^U] = [(A)_\alpha^L \bullet (B)_\alpha^L, (A)_\alpha^U \bullet (B)_\alpha^U]$.

Division: $(A)_\alpha (:) (B)_\alpha = [(A)_\alpha^L, (A)_\alpha^U] (:) [(B)_\alpha^L, (B)_\alpha^U] = [(A)_\alpha^L / (B)_\alpha^U, (A)_\alpha^U / (B)_\alpha^L]$.

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