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Inverse coefficient problems for a first order hyperbolic system

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ABSTRACT

In this paper, the direct and inverse initial boundary value problems for a first order system of two hyperbolic equations are considered. The method of characteristics and the finite difference method are applied to the theoretical and numerical solutions of the direct problem, respectively. Moreover the suitability of the method of characteristics for the inverse problem of finding solely space-dependent coefficients and the finite difference method for solely time-dependent coefficients of the first order hyperbolic system are shown. The stability of the numerical method is supported by the examples.

1. Introduction

Certain problems arising in physics and engineering are modeled by hyperbolic initial-boundary value problems (IBVPs). The IBVPs for the first order system of hyperbolic partial differential equations (PDEs) have a special place in the mathematical literature. Problems of this category appear in many mathematical models including electric oscillation in a transmission line, small vibration of a string with damping term, problems in gas dynamics and acoustic wave propagation [8,11]. In the past few years the interest to the solutions of these problems has increased [21,27,28].

In the domain $D = \{(x,t): \ 0 < x < \ell, \ t > 0\}$, we consider the hyperbolic system of two equations

$$\begin{aligned} &\frac{\partial v}{\partial t} - \frac{\partial v}{\partial x} + q(x,t)u = 0, \\ &\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + p(x,t)v = 0, \end{aligned} \tag{1}$$

with the boundary conditions

$$u\,|_{x=0} = g_1(t), \ \ v\,|_{x=\ell} = g_2(t), \ \ t \ge 0, \tag{2}$$

and the initial conditions

$$u\,|_{t=0} = \psi(x), \ \ v\,|_{t=0} = \varphi(x), \ \ 0 \le x \le \ell. \tag{3}$$

When the coefficients $p(x,t)$ and $q(x,t)$ are known, the initial-boundary value problem (IBVP) of finding $\{u(x,t), v(x,t)\}$ from the system (1), boundary conditions (2) and initial conditions (3) is termed as the direct (or forward) problem. The forward problem for the system (1) is well known from the literatures [11,16,22].

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When the functions $p(x,t)$ and $q(x,t)$ are unknown, the inverse IBVP formulates as a problem of finding $\{u(x,t), v(x,t)\}$ together with $\{p(x,t), q(x,t)\}$ such that they satisfy the system (1), boundary conditions (2), initial conditions (3) and some over-determination conditions. The inverse IBVP for the first order hyperbolic system when the coefficients depends both time and space variables is not studied neither theoretically nor numerically. It is important to notice the papers [6,7,23] and [14,19,24] where the inverse Cauchy and inverse scattering problems are studied for the general first order hyperbolic system, respectively. Our goal is to study the inverse IBVP in the cases when coefficients are dependent solely on one variable. If the over-determination conditions are

$$u\,|_{x=\ell}= h_1(t),\ \ v\,|_{x=0}= h_2(t),\ t \geq 0 \tag{4}$$

it will be seen that the characteristics method and the finite difference method are suitable for the inverse problem (1)–(4) in the case of solely space and solely time dependent coefficients, respectively.

When the coefficients are dependent solely on one variable, the various inverse IBVPs for the system (1) are considered theoretically in [9,21,22,27–29] and numerically in [1,3,15,26].

First of all, notice the reference [22], where the inverse IBVP is studied for the system (1) with the matrix coefficients $\begin{bmatrix} r(x) & p(x) \\ q(x) & s(x) \end{bmatrix}$ from two collection of boundary (2), initial (3) and over-determination (4) conditions, by using the method of characteristics. The techniques of this method are applied to the inverse IBVP for the system (1) with self-adjoint, space-dependent coefficients in the half-plane $x \geq 0$ where the additional condition is the second condition in (4) [21].

The invariant imbedding method of investigation the IBVP of finding the concentration parameter in the coefficients for the system (1) with the space-dependent coefficients in the half-plane $x \geq 0$ with the additional condition $v\,|_{x=0}= h_2(t)$, $t \geq 0$ is considered in [9] and the generalization to matrix case is studied in [3].

The simultaneous determination of some components of space-dependent coefficients of the first order hyperbolic system of two equation and its $2n$ matrix generalization with the boundary conditions (2) and (4) are studied in papers [27,28] by using spectral properties and transformation operator's techniques of the auxiliary spectral problem. The Gelfand–Levitan theory is used in the inverse IBVP for the system (1) with the space-dependent coefficients in the case $q(x) = -p(x)$ from the impulse response $u(0,t) = v(0,t) = h_2(t)$, $t \geq 0$ in [24].

For similar hyperbolic inverse problems, see [8,16,20] and references therein.

The finite difference method for the inverse IBVP for the system (1) is not well-known because there are limited number of papers on that problems. Since the finite difference method is not suitable for the inverse IBVP for the system (1) with the space-dependent coefficients, the finite difference method is not directly applied to the problems in the existence papers. This method is applied to the integral equation which is equivalent to the investigated inverse problems [1,3,15,26]. In present paper we show that the finite difference method can be directly applied to the inverse IBVP for the system (1) with time-dependent coefficients.

The work [13] considers the inverse problem for the wave equation which consists in determining an unknown time-dependent force function by applying finite difference method. Same method is used for an unknown space-dependent force function acting on a vibrating structure in the wave equation from Cauchy boundary data in [12]. The other works where not only finite difference, but also another numerical methods (adaptive hybrid finite element/finite difference and globally convergent methods) are used for reconstruction of a space-dependent coefficient in hyperbolic PDE in [4,5,10].

The paper organized as follows: In Section 2, the direct IBVP for the system (1) with the both space and time-dependent coefficients is studied theoretically and numerically by using method of characteristics and finite difference method, respectively. In Section 3, the inverse IBVPs are studied for the system (1) with solely space-dependent and solely time-dependent coefficients by using method of characteristics and finite difference method, respectively.

2. Direct problem

The well-posedness of the direct problem for the more general first order hyperbolic system with the strictly dissipative boundary conditions are shown in [11] by the method of nets. This method also solves the direct problem numerically. In this section, we will show the existence and uniqueness of the solution of the direct problem by using method of characteristics. We will also solve this problem numerically, by applying finite difference method. The reason of choosing these methods is to adapt them to inverse problems for the system (1) with the space-dependent and time-dependent coefficients, respectively. Briefly, this section includes the facts which are auxiliary for the next section.

2.1. Method of characteristics

In this subsection, we shall examine the direct problem (1)–(3) in the domain D under the following consistency conditions between the initial data and the boundary data:

$$\psi(0) = g_1(0),\ \varphi(\ell) = g_2(0). \tag{5}$$

The characteristics outgoing from the terminal ends of the intercept $(0,\ell)$ of the *x-axis* to the right and left boundaries of D form 4 domains as follows: