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Finite element solution of nonlinear eddy current problems with periodic excitation and its industrial applications

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ABSTRACT

An efficient finite element method to take account of the nonlinearity of the magnetic materials when analyzing three-dimensional eddy current problems is presented in this paper. The problem is formulated in terms of vector and scalar potentials approximated by edge and node based finite element basis functions. The application of Galerkin techniques leads to a large, nonlinear system of ordinary differential equations in the time domain.

The excitations are assumed to be time-periodic and the steady-state periodic solution is of interest only. This is represented either in the frequency domain as a finite Fourier series or in the time domain as a set of discrete time values within one period for each finite element degree of freedom. The former approach is the (continuous) harmonic balance method and, in the latter one, discrete Fourier transformation will be shown to lead to a discrete harmonic balance method. Due to the nonlinearity, all harmonics, both continuous and discrete, are coupled to each other.

The harmonics would be decoupled if the problem were linear, therefore, a special nonlinear iteration technique, the fixed-point method is used to linearize the equations by selecting a time-independent permeability distribution, the so-called fixed-point permeability in each nonlinear iteration step. This leads to uncoupled harmonics within these steps.

As industrial applications, analyses of large power transformers are presented. The first example is the computation of the electromagnetic field of a single-phase transformer in the time domain with the results compared to those obtained by traditional time-stepping techniques. In the second application, an advanced model of the same transformer is analyzed in the frequency domain by the harmonic balance method with the effect of the presence of higher harmonics on the losses investigated. Finally a third example tackles the case of direct current (DC) bias in the coils of a single-phase transformer.

1. Introduction

The most straightforward method of solving nonlinear electromagnetic field problems in the time domain by the method of finite elements (FEM) is using time-stepping techniques. This requires the solution of a large nonlinear equation system at each time step and is, therefore, very time consuming, especially if a three-dimensional problem is being treated. If the excitations are non-periodic or if, in case of periodic excitations, the transient solution is required, one cannot avoid time stepping. In many cases however, the excitations of the problem are periodic, and it is only the steady-state periodic solution

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which is needed. Then, it is wasteful to step through several periods to achieve this by the "brute force" method [1] of time stepping.

A successful method to avoid stepping through several periods in such a case is the time-periodic finite element method introduced in [12]. To accelerate the originally slow convergence of the method a singular-decomposition technique has been introduced in [18] and it has even been parallelized in [19].

A new time domain technique using the fixed-point method to decouple the time steps has been introduced in [9] and applied to two-dimensional eddy current problems described by a single component vector potential. The optimal choice of the fixed-point permeability for such problems has been presented in [13] both in the time domain and using harmonic balance principles. The method has been applied to three-dimensional problems in terms of a magnetic vector potential and an electric scalar potential ($\boldsymbol{A}, V$–$\boldsymbol{A}$ formulation) in [14] and, employing a current vector potential and a magnetic scalar potential ($\boldsymbol{T}, \Phi$–Φ formulation), in [15] and [8]. In contrast to the time-periodic finite element method, the periodicity condition is directly present in the formulation instead of being satisfied iteratively.

The aim of this work is to present a detailed review of the fixed-point based method and to show its application to industrial problems arising in the design of large power transformers.

The paper is structured as follows: In the following two sub-sections of the Introduction, two FEM potential formulations of eddy current problems are briefly reviewed and the continuous and discrete harmonic balance methods to obtain their steady-state periodic solution are introduced. In Section 2, a method is developed to decouple the harmonics from each other and hence to solve for each harmonic separately. This is trivial for linear problems, but a special fixed-point iteration technique is introduced to treat nonlinearity with the harmonics decoupled. Section 3 is devoted to numerical examples involving large power transformers. The results of the paper are concluded in Section 4.

1.1. Finite element potential formulations

The geometry of an eddy current problem can be naturally split in two: an eddy current domain with unknown current density distribution and an eddy current free region in which the current density is given [3].

The electromagnetic field problem to be solved in the eddy current domain Ω_c consisting of conducting media is described by Maxwell's equations in the quasi-static limit:

$$curl\,\boldsymbol{H} = \boldsymbol{J} + curl\,\boldsymbol{T}_0, \tag{1}$$

$$curl\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{2}$$

$$div\,\boldsymbol{B} = 0, \tag{3}$$

$$div\,\boldsymbol{J} = 0 \tag{4}$$

where $\boldsymbol{H}$ is the magnetic field intensity, $\boldsymbol{J}$ is the eddy current density, $\boldsymbol{T}_0$ is an impressed current vector potential whose curl is the given current density in coils external to Ω_c, $\boldsymbol{E}$ is the electric field intensity, $\boldsymbol{B}$ is the flux density and t is time. In the eddy current free region Ω_n (such as domains containing non-conducting media as well as coils with known current density) it is sufficient to solve (1) with $\boldsymbol{J} = 0$ in addition to (3) for the magnetic field quantities. The material relationships are

$$\boldsymbol{B} = \mu\big(|\boldsymbol{H}|\big)\boldsymbol{H} \quad \text{or} \quad \boldsymbol{H} = \nu\big(|\boldsymbol{B}|\big)\boldsymbol{B}, \tag{5}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} \quad \text{or} \quad \boldsymbol{E} = \rho \boldsymbol{J} \tag{6}$$

where μ is the permeability, ν is its reciprocal, the reluctivity and σ is the conductivity with ρ denoting its reciprocal, the resistivity. In magnetic materials (steel), the relationships (5) are nonlinear, i.e. the permeability and the reluctivity depend on the magnetic field intensity or the magnetic flux density as indicated.

The numerical solution of the problem is carried out by the method of finite elements. The application of FEM is straightforward if potential functions are introduced. Basically, two options are open: the field quantities can either be represented by a magnetic vector potential $\boldsymbol{A}$ and an electric scalar potential V ($\boldsymbol{A}, V$–$\boldsymbol{A}$ formulation) as

$$\boldsymbol{B} = curl\,\boldsymbol{A} \quad \text{in } \Omega_c \cup \Omega_n, \qquad \boldsymbol{E} = -\frac{\partial}{\partial t}(\boldsymbol{A} + grad\,V) \quad \text{in } \Omega_c, \tag{7}$$

or by a current vector potential $\boldsymbol{T}$ and a magnetic scalar potential Φ ($\boldsymbol{T}, \Phi$–Φ formulation) as

$$\boldsymbol{H} = \boldsymbol{T}_0 + \boldsymbol{T} - grad\,\Phi \quad \text{in } \Omega_c \cup \Omega_n, \qquad \boldsymbol{J} = curl\,\boldsymbol{T} \quad \text{in } \Omega_c \tag{8}$$

with $\boldsymbol{T} = 0$ in Ω_n. The definitions (7) satisfy (2) and (3), whereas those in (8) ensure that (1) and (4) hold. Therefore, the differential equations (1) and (4) are to be solved in the $\boldsymbol{A}, V$–$\boldsymbol{A}$ formulation:

$$curl(\nu\,curl\,\boldsymbol{A}) + \frac{\partial}{\partial t}\big[\sigma(\boldsymbol{A} + grad V)\big] = curl\,\boldsymbol{T}_0, \tag{9}$$

$$-div\left[\sigma \frac{\partial}{\partial t}(\boldsymbol{A} + grad\,V)\right] = 0, \tag{10}$$