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Time domain CFIEs for electromagnetic scattering problems

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ABSTRACT

Time domain integral equations complement other methods for solving Maxwell's equations by handling infinite domains without difficulty and by reducing the computational domain to the surface of the scatterer. In this paper we study the discretization error when convolution quadrature is used to discretize two new regularized combined field integral equation formulations of the problem of computing scattering from a bounded perfectly conducting obstacle.

1. Introduction

We shall derive error estimates for approximating a time domain electromagnetic scattering problem via two new regularized time domain combined field integral equations (CFIEs). Time domain integral equations are gaining popularity for solving electromagnetic scattering problems because they handle infinite domains easily and fast solution techniques are available [31,1,19,30]. In addition they reduce the computational domain to the boundary of the scatterer (the advantage of this dimension reduction by solving only on the boundary of the domain is less evident in the time domain because of the need to store a history of the solution there). The standard method for discretizing these equations is a space–time Galerkin method (e.g. [31] for the electric field integral equation case), but then retarded potential integrals need careful evaluation which complicates the use of high order and curvilinear elements. As an alternative we shall analyze the use of convolution quadrature (CQ) to discretize in time [27]. This has already been used to discretize the related time domain electric field integral equation and provides a stable numerical time marching scheme [4,27,34] that can be used for curvilinear and higher order elements [34].

We wish to approximate the electromagnetic field scattered by a bounded metallic object which we model by a perfectly electrically conducting (PEC) boundary condition. The scatter is assumed to occupy an open bounded Lipschitz polyhedral domain having connected complement and denoted by $\Omega \subset \mathbb{R}^3$. Let $\Gamma := \partial\Omega$ be the boundary of Ω and $\mathbf{n}$ be the unit outward normal on the boundary Γ. The unbounded exterior of Ω is denoted by $\Omega^+ := \mathbb{R}^3 \backslash \overline{\Omega}$. The domain Ω^+ outside the scatterer is assumed to be free space and so is homogeneous and isotropic. Then, denoting by $T > 0$ the final time of the calculation, the time domain electric and magnetic scattered fields $(\mathscr{E}, \mathscr{H})$ satisfy the Maxwell system

$$\varepsilon_0 \frac{\partial \mathscr{E}}{\partial t} - \operatorname{curl} \mathscr{H} = \mathbf{0}, \quad \text{in } \Omega^+ \times (0, T], \tag{1.1a}$$

$$\mu_0 \frac{\partial \mathscr{H}}{\partial t} + \operatorname{curl} \mathscr{E} = \mathbf{0}, \quad \text{in } \Omega^+ \times (0, T], \tag{1.1b}$$

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$$\mathscr{E} \times \mathbf{n} = \mathscr{G}, \quad \text{on } \Gamma \times (0, T], \tag{1.1c}$$

$$\mathscr{E}(\cdot, t) = \mathbf{0}, \quad \text{in } \Omega^+ \text{ for } t \leqslant 0, \tag{1.1d}$$

$$\mathscr{H}(\cdot, t) = \mathbf{0}, \quad \text{in } \Omega^+ \text{ for } t \leqslant 0, \tag{1.1e}$$

where the boundary data $\mathscr{G}$ is given in terms of the incident field $\mathscr{E}^{inc}$ as follows. The incident field is assumed to be a smooth solution of (1.1a)–(1.1b) in a neighborhood of $\overline{\Omega}$ that vanishes in a neighborhood of Γ for $t \leqslant 0$. Then $\mathscr{G} = -\mathscr{E}^{inc} \times \mathbf{n}$ on Γ because of the assumed PEC boundary condition on the scatterer. The constants ε_0 and μ_0 are the permittivity and permeability for free space respectively.

After a change of variables (or by choosing appropriate units) [18], we can assume $\varepsilon_0 = 1$ and $\mu_0 = 1$. By taking the time derivative of the electric equation and using the magnetic equation, we can rewrite Eqs. (1.1) in terms of electric scattered field alone as follows:

$$\frac{\partial^2 \mathscr{E}}{\partial t^2} + \operatorname{curl}\operatorname{curl}\mathscr{E} = \mathbf{0}, \quad \text{in } \Omega^+ \times (0, T], \tag{1.2a}$$

$$\mathscr{E} \times \mathbf{n} = \mathscr{G}, \quad \text{on } \Gamma \times (0, T], \tag{1.2b}$$

$$\mathscr{E}(\cdot, 0) = \frac{\partial \mathscr{E}}{\partial t}(\cdot, 0) = \mathbf{0}, \quad \text{in } \Omega^+. \tag{1.2c}$$

The focus of this paper is on solving the time domain Maxwell equations (1.2) by time domain boundary integral equation (TD-BIE) methods.

To derive a standard classical TD-BIE, the following time domain single layer ansatz is often assumed for the solution $\mathscr{E}$ of problem (1.2):

$$\mathscr{E}(\mathbf{x}, t) = \int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\eta_t(\mathbf{y}, \tau)\, d\tau\, d\sigma_y - \nabla \int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\, \operatorname{div}_\Gamma\left(\int_0^\tau \eta(\mathbf{y}, \gamma)\, d\gamma\right) d\tau\, d\sigma_y, \tag{1.3}$$

for $\mathbf{x} \notin \Gamma$, and for some unknown tangential boundary field η and where $\operatorname{div}_\Gamma$ denotes the surface divergence. Here the kernel k is the fundamental solution of the wave equation given by

$$k(\mathbf{x}, t) := \frac{\delta(t - |\mathbf{x}|)}{4\pi |\mathbf{x}|}, \quad \mathbf{x} \neq \mathbf{0},$$

with δ denoting the delta distribution. Proceeding formally, the field η can then be related to the boundary data $\mathscr{G}$ by taking the tangential component of the trace of the above representation (1.3) on Γ. Let Π_T denote the tangential projection on Γ, given by

$$\Pi_T \mathbf{v} = \mathbf{v}_T := \mathbf{n} \times (\mathbf{v} \times \mathbf{n}) = \mathbf{v} - (\mathbf{v} \cdot \mathbf{n})\mathbf{n},$$

and let ∇_Γ denote the surface gradient. Then using the continuity of the tangential trace of the single layer for Maxwell's equations (under suitable conditions on η and Γ) we obtain the following time domain electric field integral equation (TD-EFIE)

$$\begin{aligned} V(\partial_t)\eta(\mathbf{x}, t) &:= \Pi_T \int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\eta_t(\mathbf{y}, \tau)\, d\tau\, d\sigma_y - \nabla_\Gamma \int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\, \operatorname{div}_\Gamma\left(\int_0^\tau \eta(\mathbf{y}, \gamma)\, d\gamma\right) d\tau\, d\sigma_y \\ &= \mathbf{n}(\mathbf{x}) \times \mathscr{G}(\mathbf{x}), \quad \mathbf{x} \in \Gamma. \end{aligned} \tag{1.4}$$

For a survey of time dependent integral equations and their application to boundary value problems including Maxwell's equations see [20].

The TD-EFIE is a popular choice when solving Maxwell's equations, but to avoid possible numerical instability [21] it is often preferred to use a combined field integral equation (CFIE). In particular, the following classical ansatz represents $\mathscr{E}$ in terms of a combination of electric and magnetic operators. We suppose there is a tangential field $\xi(\mathbf{x}, t)$ such that the solution $\mathscr{E}$ of problem (1.2) has the form, for $\mathbf{x} \notin \Gamma$,

$$\begin{aligned} \mathscr{E}(\mathbf{x}, t) = &\int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\xi_t(\mathbf{y}, \tau)\, d\sigma_y\, d\tau - \nabla \int_0^t \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\, \operatorname{div}_\Gamma\left(\int_0^\tau \xi(\mathbf{y}, \gamma)\, d\gamma\right) d\sigma_y\, d\tau \\ &+ \int_0^t \operatorname{curl} \int_\Gamma k(\mathbf{x} - \mathbf{y}, t - \tau)\left(\int_0^\tau \xi(\mathbf{y}, \gamma)\, d\gamma\right) d\sigma_y\, d\tau, \end{aligned} \tag{1.5}$$