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The Orthogonal Art Gallery Theorem with Constrained Guards

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Abstract

Let P be an orthogonal polygon with n vertices, and let V^* and E^* be specified sets of vertices and edges of P. We prove that P has a guard set of cardinality at most $\lfloor (n + 3|V^*| + 2|E^*|)/4 \rfloor$ that includes each vertex in V^* and at least one point of each edge in E^*. Our bound is sharp and reduces to the orthogonal art gallery theorem of Kahn, Klawe and Kleitman when V^* and E^* are empty.

Keywords: art gallery theorem, orthogonal polygon, graph coloring, polygon visibility.

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1 Introduction: Art Galleries with Constrained Guards

The original art gallery problem, posed by Klee in 1973, asks for the minimum number of guards (points) sufficient to protect any simple polygon.(the art gallery). Every point inside the polygon must be visible to at least one guard, i.e., the segment joining the point to some guard must not intersect the exterior of the polygon. The first solution to this problem was given by Chvátal [1], who proved that $\lfloor n/3 \rfloor$ guards are sometimes necessary, and always sufficient to cover a polygon with n vertices. Later Fisk [2] provided a shorter proof of Chvátal's theorem using an elegant graph coloring argument. Klee's art gallery problem has since grown into a significant area of study. Numerous variants of Klee's art gallery problem have been proposed and studied with different restrictions placed on the shape of the galleries or the powers of the guards. (See the monograph by O'Rourke [6], and the surveys by Shermer [8] and Urrutia [7].)

Some of the most important variants of Klee's art gallery problem deal with *orthogonal polygons* — those whose interior angles are 90° or 270°. In 1983, Kahn, Klawe and Kleitman [3] proved the fundamental theorem about orthogonal art galleries:

Theorem 1.1 ([3]) *An orthogonal art gallery with n vertices can be protected by $\lfloor n/4 \rfloor$ vertex guards.*

A very recent variant of Klee's art gallery problem [5] deals with constraints on the placement of the guards in the triangulation graph $G = (V, E)$. Each vertex of a specified subset V^* must be a guard, and each edge of a specified edge set E^* must contain a guard. Here is the main theorem of [5]:

Theorem 1.2 ([5]) *Let P be a polygon with n vertices. If V^* and E^* are specified vertex and edge subsets of P, then P has a guard set of cardinality at most*

$$\left\lfloor \frac{n + 2|V^*| + |E^*|}{3} \right\rfloor$$

that includes each vertex in V^* and at least one point from each edge in E^*.

The ordinary art gallery theorem is the special case in which V^* and E^* are empty.

Our main theorem establishes the corresponding result for orthogonal polygons:

Theorem 1.3 *Let P be an orthogonal polygon with n vertices. If V^* and E^* are specified vertex and edge subsets of P, then P has a guard set of cardinality*