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Independent $[1,2]$-domination of grids via min-plus algebra

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Abstract

Domination of grids has been proved to be a demanding task and with the addition of independence it becomes more challenging. It is known that no grid with $m,n \geq 5$ has a perfect code, that is an independent vertex set such that each vertex not in it has exactly one neighbor in that set. So it is interesting to study the existence of an independent dominating set for grids that allows at most two neighbors, such a set is called independent $[1,2]$-set. In this paper we develop a dynamic programming algorithm using min-plus algebra that computes the minimum cardinality of an independent $[1,2]$-set for the grid $P_m \Box P_n$.

Keywords: Domination, independence, grids, min-plus algebra.

1 Introduction

Let $G = (V,E)$ be a simple graph. A subset $S \subseteq V$ is called a dominating set of G if every $v \in V \setminus S$ has at least one neighbor in S. Recall that the grid $P_m \Box P_n$ is the cartesian product of paths P_m and P_n. Domination in grids

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has been extensively studied and the problem of determining the domination number $\gamma(P_m \Box P_n)$, which is the minimum size of a dominating set of $P_m \Box P_n$ was opened for almost 30 years, since it was first studied in [6]. In his 1992 Ph.D thesis Chang [1] proved that the domination number in grids is bounded by $\gamma(P_m \Box P_n) \leq \lfloor \frac{(n+2)(m+2)}{5} \rfloor - 4$, for $m, n \geq 8$. Chang also conjectured that equality holds for $16 \leq m \leq n$. In 1998 $\gamma(P_m \Box P_n)$ was computed for $m \leq 19$ and every n (see [8]). Finally in 2011, the problem was completely solved in [4] as authors were able to adapt the ideas in [5] to confirm Chang′s conjecture.

Independence is a property closely related to domination. A set S of vertices is called *independent* if no two vertices in S are adjacent. The independent domination number is denoted by $i(G)$, which is the minimum cardinality of an independent dominating set for the graph G. Recently in [3] the independent domination number has been computed for all grids.

A *perfect code* is an independent dominating set such that every vertex not in it has a unique neighbor in the set. It was proved in [7] that there exists no perfect code for grids $P_m \Box P_n$, except in cases $m = n = 4$ and $m = 2, n = 2k + 1$. This leads us to work with independent $[1, 2]$-sets, that is an independent vertex set S such that every $v \in V \setminus S$ is adjacent to at least one but not more than 2 vertices in S (see [2]). We solve the open problem proposed in [2] about the existence of independent $[1, 2]$-set in grids and we also compute the independent $[1, 2]$-number $i_{[1,2]}(P_m \Box P_n)$ which is the minimum cardinality of such a set.

2 Algorithm

We present a dynamic programming algorithm to obtain $i_{[1,2]}(P_m \Box P_n)$, following the ideas in [3,8]. Consider $P_m \Box P_n$ as an array with m rows, n columns and vertex set $\{v_{ij} \colon 1 \leq i \leq m,\ 1 \leq j \leq n\}$. Let S be an independent $[1, 2]$-set of $P_m \Box P_n$. We define a labeling of vertices of $P_m \Box P_n$ associated to S as follows

$$l(v_{ij}) = \begin{cases} 0 & \text{if } v_{ij} \in S \\ 1 & \text{if } v_{ij} \notin S \text{ and } |\{v_{i(j-1)}, v_{(i-1)j}, v_{(i+1)j}\} \cap S| = 1 \\ 2 & \text{if } v_{ij} \notin S \text{ and } |\{v_{i(j-1)}, v_{(i-1)j}, v_{(i+1)j}\} \cap S| = 2 \\ 3 & \text{if } v_{ij} \notin S \text{ and } |\{v_{i(j-1)}, v_{(i-1)j}, v_{(i+1)j}\} \cap S| = 0 \end{cases}$$

Given an independent $[1, 2]$-set, we can identify each vertex with its label so we obtain an array of labels with m rows and n columns. Hereinafter, given an independent $[1, 2]$-set of $P_m \Box P_n$, the columns of the grid are words