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Bounds on spectrum graph coloring

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Abstract

We propose two vertex-coloring problems for graphs, endorsing the spectrum of colors with a matrix of interferences between pairs of colors. In the THRESHOLD SPECTRUM COLORING problem, the number of colors is fixed and the aim is to minimize the maximum interference at a vertex (interference threshold). In the CHROMATIC SPECTRUM COLORING problem, a threshold is settled and the aim is to minimize the number of colors (among the available ones) for which respecting that threshold is possible. We prove general upper bounds for the solutions to each problem, valid for any graph and any matrix of interferences. We also show that both problems are NP-hard and perform experimental results, proposing a DSATUR-based heuristic for each problem, in order to study the gap between the theoretical upper bounds and the values obtained.

Keywords: graph coloring, threshold spectrum coloring, chromatic spectrum coloring, DSATUR, frequency assignment, WiFi channel assignment

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1 Introduction

Graph coloring is one of those problems in Discrete Mathematics appealing both mathematicians and engineers [6,8], with frequency assignment being one of its prominent applications [1]. Inspired by WiFi channel assignment, we consider an abstract undirected graph G together with a spectrum of colors $S = \{c_1, \ldots, c_s\}$ (representing available channels) for which we have a symmetric matrix W of non-negative distances $W_{ij} = W(c_i, c_j)$ between each pair of colors (representing interferences between each pair of channels). Thus, a coloring c of the graph induces an *interference* at each vertex v:

$$I_v(G, W, c) = \sum_{u \in N(v)} W(c(u), c(v)).$$

In our work [4] we have applied succesfully this model to find efficient frequency assignments in Wireless Networks. In the present work, we introduce the Threshold Spectrum Coloring (TSC) problem, which considers a graph G and a spectrum of k colors endorsed with a $k \times k$ matrix W of interferences between them, with the goal of determining the minimum threshold $t \in \mathbb{R}_{\geq 0}$ such that (G, W) admits a k-coloring c in which the interference at every vertex is at most t, i.e., $I_v(G, W, c) \leq t$, $\forall v$. Such a minimum t will be called the *minimum k-chromatic threshold* of (G, W), denoted as $T_k(G, W)$.

We also introduce the counterpart Chromatic Spectrum Coloring (CSC) problem, which considers a threshold $t \in \mathbb{R}_{\geq 0}$, letting the size of the spectrum to be the number $|V(G)|$ of vertices, with the goal of determining the minimum number of colors $k \in \mathbb{N}$ such that (G, W) admits a k-coloring c in which the interference at every vertex is at most that threshold t. Such a minimum k will be called the *t-interference chromatic number* of (G, W), denoted as $\chi_t(G, W)$.

Figure 1 illustrates the TSC problem for the paw graph PG with a spectrum of $k = 3$ colors endorsed with a matrix W_{2ed} in which interferences decay exponentially with base 2, leading to $T_3(PG, W_{2ed}) = 1$. For the CSC problem, the corresponding 4×4 matrix and interference threshold $t = 1$ lead to $\chi_1(PG, W_{2ed}) = 3$.

As for related work, Araujo et al. [2] consider a weight function w on the edges of the graph G, instead of a matrix W of interferences between colors, defining the interference at a vertex to be the sum of weights of incident monochromatic edges. Many other works impose conditions to the colors of the endpoints of any edge, many of which can be framed into $L(p_1, \ldots, p_k)$-*labellings* [3], where vertices at distance i must get colors at distance $\geq p_i$.

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