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Vertex-disjoint cycles in bipartite tournaments

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Abstract

Let $k \geq 2$ be an integer. Bermond and Thomassen [Bermond J. C., Thomassen, C., Cycles in digraphs a survey, Journal of Graph Theory 5(1) (1981) 1–43] conjectured that every digraph D with $\delta^+(D) \geq 2k-1$ contains at least k vertex-disjoint cycles. In this work we prove that every bipartite tournament with minimum out-degree at least $2k-2$ and minimum in-degree at least one contains k vertex-disjoint cycles of length four, whenever $k \geq 3$. Finally, we show that every bipartite tournament with minimum degree at least $(3k-1)/2$ contains k vertex-disjoint cycles of length four.

Keywords: vertex-disjoint cycles, bipartite tournament, minimum degree

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1 Introduction

Bermond and Thomassen [4], in 1981, stablished the following conjecture, which relates the number of disjoint cycles with the minimum out-degree of a digraph.

Conjecture 1.1 *Every digraph with D with $\delta^+(D) \geq 2k-1$ contains at least k vertex-disjoint cycles.*

Thomassen [7] established the existence of a finite integer $f(k)$ such that every digraph of minimum out-degree at least $f(k)$ contains k disjoint cycles. In 1996, Alon [1] proved that $f(k) \leq 64k$ for every positive integer k. Conjecture 1.1 has been proved, by Thomassen [7], when $k = 2$, and by Lichiardopol, Pór and Sereni [6] if $k = 3$. In 2010, Bessy, Lichiardopol and Sereni [3] proved it for regular tournaments. In 2014, Bang-Jensen, Bessy and Thomassé [5] proved the conjecture for tournaments. Recently, Bay, Li and Li [2] proved Conjecture 1.1 for bipartite tournaments.

By considering the girth of a digraph Bang Jensen, Bessy and Thomassen [3] proposed the following conjecture.

Conjecture 1.2 *Every digraph D with girth $g \geq 2$ and $\delta^+(D) \geq gk/(g-1)$ contains at least k vertex-disjoint cycles.*

In this work we prove that every bipartite tournament T with $\delta^+(T) \geq 2k-2$ and $\delta^-(T) \geq 1$ contains at least k vertex-disjoint cycles of length four. It is also shown that every bipartite tournament T with $\delta(T) \geq (3k-1)/2$ contains at least k vertex-disjoint cycles of length four. As a consequence, it is proved that Conjecture 1.2 holds for bipartite tournaments with minimum in-degree at least one and $k = 2, 3, 4$.

2 Basic definitions

Through this work only finite digraphs without loops and multiple arcs are considered. Let D be a digraph with vertex set $V(D)$ and arc set $A(D)$. The *out-degree* of a vertex u of a digraph D is the number of arcs exiting from u. The *in-degree* of u is the number of arcs entering into u. These integers are denoted by $d^+(u)$ or $d^-(u)$, respectively. We denote by $\delta^+(D)$ the minimum out-degree of the vertices in D, and by $\delta^-(D)$ the minimum in-degree of the vertices in D. The *minimum degree* is defined as $\delta(D) = \min\{\delta^+(D), \delta^-(D)\}$. The *girth* of a digraph D is the minimum length of a cycle in D. A *tournament* is an orientation of a complete graph and a *bipartite tournament* is an oriented