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Highly Connected Subgraphs of Graphs with Given Independence Number (Extended Abstract)

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Abstract

Let G be a graph on n vertices with independence number α. What is the largest k-connected subgraph that G must contain? We prove that if n is sufficiently large ($n \geq \alpha^2 k + 1$ will do), then G contains a k-connected subgraph on at least n/α vertices. This is sharp, since G might be the disjoint union of α equally-sized cliques. For $k \geq 3$ and $\alpha = 2, 3$, we shall prove that the same result holds for

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$n \geq 4(k-1)$ and $n \geq \frac{27(k-1)}{4}$ respectively, and that these lower bounds on n are sharp.

Keywords: Connectivity, highly connected subgraph, independence number

1 Introduction

For any terms not defined here, we refer the reader to [1]. When can we find a large highly connected subgraph of a given graph G? A classical theorem due to Mader [10] (see also [5]) states that if G has average degree at least $4k$, then G contains a k-connected subgraph H. Mader's theorem does not give a lower bound on the order of H. If G is dense (for instance if $\delta(G)$, the minimum degree of G, is bounded below), it is natural to expect that G in fact contains a *large* highly connected subgraph. Using a recent result of Borozan et al. [3], we know that every graph G of order n with $\delta(G) \geq \sqrt{c(k-1)n}$ contains a k-connected subgraph of order at least $\sqrt{(k-1)n/c}$, where $c = 2123/180$. What if we are interested in finding a larger k-connected subgraph, say of order cn? Along these lines, Bollobás and Gyárfás [2] conjectured that any graph G of order $n \geq 4k-3$, or its complement $\overline{G}$, contains a k-connected subgraph H of order at least $n - 2(k-1)$. Since either G or $\overline{G}$ is a dense graph, we might expect to find a very large highly connected subgraph in one of them. This conjecture was settled affirmatively for $n \geq 13k - 15$ by Liu, Morris and Prince [9], and then for $n > 6.5(k-1)$ by Fujita and Magnant [8].

Suppose next that $\delta(G) = cn$. Can we find a k-connected subgraph of G on at least cn vertices? It turns out that the answer is "yes" for sufficiently large n, and in fact a simple argument gives even more. To see this, suppose $n \gg k$ and $n \gg 1/c$, and let $m = \lfloor 1/c \rfloor$. If G itself is not k-connected, then G can be "split" into two pieces with a (negligible) separating set of size at most $k-1$. Both pieces must have order at least cn, so as not to violate the minimum degree condition. Discard one of the pieces, together with the separating set, to obtain a new graph G'. If G' is not k-connected, we continue the process, which terminates after at most $m-1$ steps, leaving a k-connected graph H on at least cn vertices. Now either this graph H, or one of the (at most $m-1$)

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