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Combinatorial bounds on connectivity for dominating sets in maximal outerplanar graphs

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Abstract

In this article we study some variants of the domination concept attending to the connectivity of the subgraph generated by the dominant set. This study is restricted to maximal outerplanar graphs. We establish tight combinatorial bounds for connected domination, semitotal domination, independent domination and weakly connected domination for any n-vertex maximal outerplaner graph.

Keywords: Domination, Maximal Outerplanar Graph, Connectivity.

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1 Introduction

Given a graph $G = (V, E)$ a *dominating set* is a set $S \subseteq V$ such that every vertex not in D is adjacent to a vertex in D. The *domination number* $\gamma(G)$ is the number of vertices in a smallest dominating set for G. In graph theory, dominanting set problems have received much attention in numerous articles and books, being the fundamental reference the book of Haynes, Hedetniemi and Slater [7], where some variants of domination are analyzed that take into account the connectivity of the subgraph generated by the dominant set. In recent years it has received special attention the problem of domination in outerplanar graphs (e.g., [1,2,11]) A graph is *outerplanar* if it has a crossing-free embedding in the plane such that all vertices are on the boundary of its outer face (the unbounded face). An outerplanar graph is *maximal* if it is not possible to add an edge such that the resulting graph is still outerplanar. A maximal outerplanar graph embedded in the plane corresponds to a triangulation of a polygon. The works mentioned above continued the work started by Matheson and Tarjan [9], where the authors proved that the domination number of a triangulated disc of order $n \geq 3$ is at most $\frac{n}{3}$. A *triangulated disc*, or triangulation graph, is a plane graph such that all its faces, except the infinite face, are triangles. In this article we establish tight combinatorial bounds for the following domination variants in maximal outerplanar graphs: connected, semitotal, weakly connected and independent. All these variants (and total domination) refers to connectivity of dominating sets. Weakly connected domination was introduced by Grossman [6] and semitotal domination by Goddard et al. [5]. The total domination variant has recently been studied, from a combinatorial point of view by Dorfling et al. [3]. They show that a maximal outerplanar graph of order $n \geq 5$ has total domination number at most $\frac{2n}{5}$, apart from two exceptions.

In the next section we describe the terminology that will be used throughout this paper and the relation between the different parameters of connectivity and domination in a maximal outerplanar graph. In sections 3 and 4 we present the obtained results for connected and independent dominations and

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