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A Dirac-type theorem for Hamilton Berge cycles in random hypergraphs

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Abstract

A Hamilton Berge cycle of a hypergraph on n vertices is an alternating sequence $(v_1, e_1, v_2, \dots, v_n, e_n)$ of distinct vertices $v_1, \dots, v_n$ and distinct hyperedges $e_1, \dots, e_n$ such that $\{v_1, v_n\} \subseteq e_n$ and $\{v_i, v_{i+1}\} \subseteq e_i$ for every $i \in [n-1]$. We prove a Dirac-type theorem for Hamilton Berge cycles in random r-uniform hypergraphs by showing that for every integer $r \geq 3$ there exists $k = k(r)$ such that for every $\gamma > 0$ and $p \geq \frac{\log^{k(r)}(n)}{n^{r-1}}$ asymptotically almost surely every spanning subhypergraph $H \subseteq H^{(r)}(n,p)$ with minimum vertex degree $\delta_1(H) \geq \left(\frac{1}{2^{r-1}} + \gamma\right) p\binom{n-1}{r-1}$ contains a Hamilton Berge cycle. The minimum degree condition is asymptotically tight and the bound on p is optimal up to possibly the logarithmic factor. As a corollary this gives a new upper bound on the threshold of $H^{(r)}(n,p)$ with respect to Berge Hamiltonicity.

Keywords: Random hypergraphs, Berge cycles, Dirac's theorem, resilience.

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1 Introduction

Many classical theorems of extremal graph theory give sufficient optimal minimum degree conditions for graphs to contain copies of large or even spanning structures. Lately it became popular to phrase such extremal results in terms of local resilience, where the *local resilience* of a graph G with respect to a given monotone increasing graph property $\mathcal{P}$ is defined as the minimum number $\rho \in \mathbb{R}$ such that one can obtain a graph without property $\mathcal{P}$ by deleting at most $\rho \cdot \deg(v)$ edges from every vertex $v \in V(G)$. For instance, using this terminology, Dirac's theorem [6] says that the local resilience of the complete graph K_n with respect to Hamiltonicity is $1/2 + o(1)$.

In recent years, an active and fruitful research direction in extremal and probabilistic combinatorics has become the study of resilience of random and pseudorandom structures. The systematic study of those with respect to various graph properties was initiated by Sudakov and Vu in [14], who in particular proved that $G(n, p)$ has resilience at least $1/2 - o(1)$ with respect to Hamiltonicity a.a.s. for $p > \log^4 n/n$. This result was improved by Lee and Sudakov [11] to $p \gg \log n/n$, which is essentially best possible with respect to both the constant $1/2$ and the edge probability, since one can find a.a.s. disconnected spanning subgraphs of $G(n, p)$ with degree at most $(1/2 - o(1))pn$ and since $G(n, p)$ itself is a.a.s. disconnected for $p \leq (1 - o(1)) \log n/n$.

A lot of resilience results are known for random graphs. For instance, the containment of triangle factors [3], almost spanning trees of bounded degree [2], pancyclic graphs [9], and almost spanning and spanning bounded degree graphs with sublinear bandwidth [1,5,8] were studied.

An *r-uniform hypergraph* is a tuple (V, E) with $E \subseteq \binom{V}{r}$ and thus the generalisation of a graph: the elements of V are called *vertices* and the elements of E *hyperedges* (or *edges* for short). It is therefore natural to ask for degree conditions that force a subhypergraph of the complete hypergraph to contain a copy of some given large structure. Such problems have been studied extensively in the last years, especially for different kinds of Hamilton cycles. Furthermore, (bounds on) the threshold for the existence of a Hamilton cycle in the random r-uniform hypergraph model $H^{(r)}(n, p)$ have been determined for various notions of cycles. We refer to [10] for an excellent survey by Kühn and Osthus of such problems.

To the best of our knowledge, there are no local resilience results for random hypergraphs at all so far. The purpose of this work is to provide a first such Dirac-type result in random hypergraphs.

We use $\deg(v)$ to denote the vertex degree of a vertex v in an r-uniform