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Random cubic planar graphs revisited

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Abstract

The goal of our work is to analyze random cubic planar graphs according to the uniform distribution. More precisely, let $\mathcal{G}$ be the class of labelled cubic planar graphs and let g_n be the number of graphs with n vertices. Then each graph in $\mathcal{G}$ with n vertices has the same probability $1/g_n$. This model was analyzed first by Bodirsky et al. [1], and here we revisit and extend their work. The motivation for this revision is twofold. First, some proofs in [1] where incomplete with respect to the singularity analysis and we aim at providing full proofs. Secondly, we obtain new results that considerably strengthen those in [1] and shed more light on the structure of random cubic planar graphs. We present a selection of our results on asymptotic enumeration and on limit laws for parameters of random graphs.

Keywords: random planar graphs, analytic combinatorics, asymptotic enumeration, limit laws

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1 Results on enumeration

Theorem 1.1 *The number* c_n *of connected cubic planar graphs with* n *vertices is asymptotically*

$$c_n \sim c \cdot n^{-7/2} \gamma^n n!,$$

with $c \approx 0.030487$ *and* $\gamma = \rho^{-1} \approx 3.132591$*, where* $\rho \approx 0.319523$ *is the smallest positive root of the equation*

$$729x^{12} + 17496x^{10} + 148716x^8 + 513216x^6 - 7293760x^4 + 279936x^2 + 46656 = 0,$$

Theorem 1.2 *The number* g_n *of cubic planar graphs with* n *vertices is asymptotically*

$$g_n \sim g \cdot n^{-7/2} \gamma^n n!,$$

where γ *is as in Theorem 1.1 and* $g \approx 0.030505$*. As a consequence, the limiting probability* p *that a random cubic planar graph is connected is equal to*

$$p = \frac{c}{g} \approx 0.999397.$$

The previous theorems were stated in [1, Theorem 2] in a less precise way and with incomplete proofs regarding the singularity analysis. Our first goal is to provide a full proof of these estimates. We remark that the actual value of p was not computed in [1]. As we will see later, p can be computed exactly using the so-called dissymmetry theorem for tree-like structures. We also remark that some of the constants given in [1] are slightly incorrect.

Theorem 1.3 *The number* h_n *of cubic planar multigraphs is asymptotically*

$$h_n \sim h \cdot n^{-7/2} \gamma_m^n n!,$$

with $h \approx 0.115965$ *and* $\gamma_m = \rho_m^{-1} \approx 3.985537$*, where* $\rho_m \approx 0.250907$ *is the smallest positive root of*

$$729x^{12} - 17496x^{10} + 148716x^8 - 513216x^6 - 7293760x^4 - 279936x^2 + 46656 = 0.$$

The same estimate holds for the number of connected cubic planar multigraphs, but with h *replaced by the constant* $h' \approx 0.104705$*. The limiting probability of connectivity is*

$$p_m = \frac{h'}{h} \approx 0.902905.$$

This result is also claimed in [3] without a detailed proof, but the equation defining ρ_m is incorrect, as well as the claimed value $\gamma_m \approx 3.973$.