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A construction of dense mixed graphs of diameter 2

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Talk dedicated to the memory of Mirka Miller

Abstract

A mixed graph is said to be *dense*, if its order is close to the Moore bound and it is *optimal* if there is not a mixed graph with the same parameters and bigger order. We give a construction that provides dense mixed graphs of undirected degree q, directed degree $\frac{q-1}{2}$ and order $2q^2$, for q being an odd prime power. Since the Moore bound for a mixed graph with these parameters is equal to $\frac{9q^2-4q+3}{4}$ the defect of these mixed graphs is $(\frac{q-2}{2})^2 - \frac{1}{4}$. In particular we obtain a known mixed Moore graph of order 18, undirected degree 3 and directed degree 1, called Bosák's graph

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and a new mixed graph of order 50, undirected degree 5 and directed degree 2, which is proved to be optimal.

Keywords: Mixed Moore graphs, projective planes.

1 Introduction and preliminaries

We consider graphs, which are finite and mixed, i.e., they may contain (directed edges) arcs as well as undirected ones. The *mixed graphs* are also called *partially directed graphs*. Bosák, in 1979, investigated those mixed graphs with given degree and given diameter having maximum number of vertices, which are called mixed Moore graphs. In some sense, Bosák generalized the concepts of Moore graph and Moore digraph by allowing the existence of both edges and arcs simultaneously. These graphs and digraphs have been much used to model different kinds of networks (such as telecommunications, multiprocessor, or local area networks, to name just a few). In many real-world networks a mixture of both unidirectional and bidirectional connections may exist (e.g. the World Wide Web network, where pages are nodes and hyperlinks describe the connections). For such networks mixed graphs provide a perfect modeling framework [2]. See [1] to check the rest of the references, by reasons of space we only give the authors and the year of publication of results.

Undirected graphs (mixed Moore graphs admitting only edges) with maximum degree d and diameter k are graphs of order $M_{d,k} = 1 + d + d(d-1) + \cdots + d(d-1)^{k-1}$ (undirected Moore bound). It is well known that there are no Moore graphs of degree $d \geq 3$ and diameter $k \geq 3$. For $k = 1$ and $d \geq 1$ complete graphs K_{d+1} are the only Moore graphs. For $k \geq 3$ and $d = 2$ the cycles C_{2k+1} are the only Moore graphs. For $k = 2$, apart from C_5 ($d = 2$), Moore graphs exist only when $d = 3$ (Petersen graph), $d = 7$ (the Hoffman-Singleton graph) and possibly $d = 57$. For more details and results concerning Moore graphs see the survey by Miller and Širáň [2].

Directed Moore graphs (mixed Moore graphs admitting only arcs) with maximum out-degree d and diameter k are digraphs of order $M^*_{d,k} = 1 + d + d^2 + \cdots + d^k$ (directed Moore bound). In 1980, it was proved that the Moore

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