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Exact value of 3 color weak Rado number

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Abstract

For integers k, n, c with k, $n \geq 1$ and $c \geq 0$, the n color weak Rado number $WR_k(n,c)$ is defined as the least integer N, if it exists, such that for every n-coloring of the set $\{1,2,...,N\}$, there exists a monochromatic solution in that set to the equation $x_1 + x_2 + ... + x_k + c = x_{k+1}$, such that $x_i \neq x_j$ when $i \neq j$. If no such N exists, then $WR_k(n,c)$ is defined as infinite.

In this work, we consider the main issue regarding the 3 color weak Rado number for the equation $x_1 + x_2 + c = x_3$ and the exact value of the $WR_2(3,c) = 13c + 22$ is established.

Keywords:
Schur numbers, sum-free sets, weak Schur numbers, weakly sum-free sets, Rado numbers, weak Rado numbers.

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1 Introduction

In terms of coloring, the Schur number $S_2(n)$ [14] is the least positive integer N such that for every n-coloring of $\{1, 2, ..., N\}$,

$\Delta : \{1, 2, ..., N\} \longrightarrow \{1, 2, ..., n\}$, there exists a monochromatic solution to the equation $x_1 + x_2 = x_3$, such that $\Delta(x_1) = \Delta(x_2) = \Delta(x_3)$ where x_1 and x_2 need not be distinct.

In 1933, Rado [9], [10] generalized the work of Schur to arbitrary systems of linear equations. Given a system of linear equations L and a natural number n, the least integer N (if it exists) such that for every coloring of the set $\{1, 2, ..., N\}$ with n colors there is a monochromatic solution to L, which is called the n color Rado number for L. If no such integer N exists, then the n color Rado number for the system L is taken to be infinite.

Eighty-three years after the first Rado results, very little progress has been obtained for some systems of linear equations. Bur and Loo [2] were able to determine the 2 color Rado number for the equations $x_1 + x_2 + c = x_3$ and $x_1 + x_2 = kx_3$ for every integer c and for every positive integer k [3].

In 1993, Schaal [12] determined the 2 color Rado number $R_k(2, c)$ for the equation $x_1 + x_2 + \ldots + x_k + c = x_{k+1}$. He also obtained [13] the 3 color Rado number $R_2(3, c)$. There are several results due to Schaal and other authors concerning 2 color and 3 color Rado numbers for particular equations, see [7], [8], [11] and other authors [6]. In addition, recently we have studied when $R_k(n, c)$ is finite or infinite and we have obtained new exacts values [1]. In this work, we consider a generalization of the Rado numbers. For every integer $c \geq 0$, $n \geq 1$, let $WR_2(n, c)$ be the least integer N (if it exists) such that, for every coloring of the set $\{1, 2, ..., N\}$ with n colors, there exists a monochromatic solution to the equation $x_1 + x_2 + c = x_3$, where $x_1 \neq x_2$. The numbers $WR_2(n, c)$ are called *weak Rado numbers*.

$WR_2(n, c)$ can be defined equivalently as the greatest N, such that the set $\{1, 2, ..., N - 1\}$ can be partitioned into n sets $\{A_1, A_2, ..., A_n\}$, such that for any $x_1, x_2 \in A_i$ then $x_1 + x_2 + c \notin A_i$, $\forall i$ where $x_1 \neq x_2$. The sets $\{A_1, A_2, ..., A_n\}$ are *weakly sum free for the equation* $x_1 + x_2 + c = x_3$.

In 1952, Walker [15] claimed the value $WR_2(5, 0) = 196$, without proof. Sixty years later, we have shown $WR_2(5, 0) \geq 196$ [4] and Schaal et al.[5] have obtained the number $WR_2(2, c)$ for every integer c.

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