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The achromatic number of Kneser graphs

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Abstract

The achromatic number α of a graph is the largest number of colors that can be assigned to its vertices such that adjacent vertices have different color and every pair of different colors appears on the end vertices of some edge.

We estimate the achromatic number of Kneser graphs $K(n,k)$ and determine $\alpha(K(n,k))$ for some values of n and k. Furthermore, we study the achromatic number of some geometric type Kneser graphs.

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1 Introduction

Let G be a finite simple graph. An *l-coloring* of G is a surjective function ς that assigns a number from the set $\{1, 2, \ldots, l\}$ to each vertex of G such that any two adjacent vertices have different colors. An l-coloring ς is called *complete* if for each pair of different colors $i, j \in \{1, 2, \ldots, l\}$ there exists an edge $xy \in E(G)$ such that $\varsigma(x) = i$ and $\varsigma(y) = j$.

While the *chromatic number* $\chi(G)$ *of* G is defined as the smallest number l for which there exists an l-coloring of G, the *achromatic number* $\alpha(G)$ *of* G is defined as the largest number l for which there exists a complete l-coloring of G (see [7]). Note that any $\chi(G)$-coloring of G is also complete. Therefore, for any graph G

$$\chi(G) \leq \alpha(G).$$

Let V be the set of all k-subsets of $\{1, 2, \ldots, n\}$, where $1 \leq k \leq n/2$. The *Kneser graph* $K(n, k)$ is the graph with vertex set V such that two vertices are adjacent if and only if the corresponding subsets are disjoint. It is well-known that $\chi(K(n, k)) = n - 2(k - 1)$ (see [5]).

A *complete geometric graph* of n points is a drawn of the complete graph K_n in the plane such that its vertices is the set P of points in general position, and its edges are straight-line segments connecting every pair of points in P. In [2], it was studied the chromatic number of graphs $D_P(n)$ whose vertex set is the set of edges of a complete geometric graph of n points and adjacency is defined in terms of geometric disjointness.

The remainder of this paper is organized as follows: In Section 2, we estimate bounds for the achromatic number of Kneser graphs. In Section 3, we determine $\alpha(K(n, 2))$ for every n. Finally, in Section 4, we study the achromatic number of graphs $D_P(n)$.

2 Bounds for $\alpha(K(n, k))$

In this section, we prove general lower and upper bounds for the achromatic number of Kneser graphs.

2.1 Upper bounds

The following upper bound for the achromatic number was proved in a particular case in [1].