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On the bipartite vertex frustration of graphs

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Abstract

The bipartite vertex (resp. edge) frustration of a graph G, denoted by $\psi(G)$ (resp. $\varphi(G)$), is the smallest number of vertices (resp. edges) that have to be deleted from G to obtain a bipartite subgraph of G. A sharp lower bound of the bipartite vertex frustration of the line graph $L(G)$ of every graph G is given. In addition, the exact value of $\psi(L(G))$ is calculated when G is a forest.

Keywords: Bipartite vertex frustration, bipartite edge frustration, line graph, Hamiltonian graph, tree.

1 Introduction

Throughout this paper, all the graphs are simple, that is, with neither loops nor multiple edges. Notations and terminology not explicitly given here can be found in the book by Chartrand and Lesniak [1].

Given a graph G with vertex set $V(G)$ and edge set $E(G)$, a subset $F \subset V(G)$ such that $G - F$ is bipartite is called a vertex bipartization for G. The

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minimum cardinality of a vertex bipartization for G is called the *bipartite vertex frustration* of G and it is denoted by $\psi(G)$. An analogous definition of *bipartite edge frustration* $\varphi(G)$ of G is stated. Thus, the bipartite vertex (resp. edge) frustration of a graph G, denoted by $\psi(G)$ (resp. $\varphi(G)$), is the smallest number of vertices (resp. edges) that have to be deleted from G to obtain a bipartite subgraph of G.

These two parameters have interesting applications in different fields of science as, for instance, fullerene chemistry. Since its very beginning, the rapid development of fullerene chemistry has been paralleled by a similarly rapid build-up of interest and a flow of results on the graphs that serve as mathematical models of fullerene isomers. Very early it became clear that the fullerene stability is related to the absence of abutting pentagons in the corresponding graphs (see, for instance, [4,6,7]). It is well known that the bipartite graphs are characterized by the absence of cycles of odd length. Hence, one may think on the number of vertices or edges that need to be removed in order to make a bipartite graph as a measure of non-bipartivity of this graph. An idea to transplant a bipartivity measure into the context of fullerene chemistry is that the minimum number of vertices and/or edges which have to be deleted to make a graph bipartite may be related to the fullerene stability.

In Graph Theory is usual to study a lot of parameters in several families of graphs. One of the best known is line graphs (see for instance [5]). The bipartite vertex/edge frustration have been also studied in several families of graphs which model different typologies of networks (see [2,3,9,10,11,12]). In [9] Yarahmadi and Ashrafi study some extremal properties of the bipartite vertex frustration of graphs and provide the exact value for the corona product of two graphs and the line graph. For this last family, there is a step in the proof of Theorem 9 of [9] which is incorrect. In this paper we find the mistake and give a sharp lower bound of the bipartite vertex frustration of the line graph. Moreover, the exact value of $\psi(L(G))$ when G is a forest is determined.

2 Main results

The *line graph* $L(G)$ of a graph G has the edge set $E(G)$ as vertex set and two vertices in $V(L(G))$ are adjacent, whenever they are incident as edges in G. In an interesting paper [9], Yarahmadi and Ashrafi study the bipartite vertex frustration of the line graph $L(G)$ of any graph G. It is understood that the considered graphs are connected, since previously, they state the following lemma whose proof is straightforward.