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On the Construction of Triples of Diagonal Latin Squares of Order 10

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Abstract

We provide a triple of diagonal Latin squares of order 10 that is the closest to being a triple of mutually orthogonal diagonal Latin squares found so far. It was obtained by constructing all orthogonal mates for diagonal Latin squares generated according to a specific scheme. We also show that a triple of mutually orthogonal diagonal

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Latin squares of order 10 cannot be constructed based on diagonal Latin squares from specific families.

Keywords: Latin square, diagonality, orthogonality, SAT.

1 Introduction

A Latin square $A = (a_{ij})$ of order n is an $n \times n$ table filled with symbols from the set $N = \{0, 1, ..., n-1\}$, in such a way that each symbol occurs precisely once in each row and each column. Diagonal Latin square is a Latin square, in which each symbol from N occurs precisely once in its main diagonal and in its main antidiagonal. Two Latin squares $A = (a_{ij})$ and $B = (b_{ij})$ are orthogonal if all ordered pairs (a_{ij}, b_{ij}) are distinct. A set of Latin squares, each two of them orthogonal, is called a set of mutually orthogonal Latin squares (MOLS). A set of diagonal Latin squares, each two of them orthogonal, is called a set of mutually orthogonal diagonal Latin squares (MODLS). A transversal of a Latin square is a set of n entries such that no two entries share the same row, column or symbol. Latin square A is isotopic to a Latin square B if B can be obtained from A by any of the following operations: permuting rows, permuting columns, permuting the names of the symbols.

Our research was mostly inspired by the papers [4] and [3]. In [4] it was determined that Latin squares of order 10 from several considered families cannot participate in a triple of MOLS of order 10. In [3] there was found the triple of Latin squares of order 10 that is the closest to being a triple of MOLS found so far. In this triple two out of three pairs of Latin squares are orthogonal, and for the third 91 pairs of elements out of 100 are distinct. We focused on obtaining similar results for diagonal Latin squares of order 10.

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