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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 54 (2016) 355–360 www.elsevier.com/locate/endm

Positive graphs

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Abstract

We call a graph positive if it has a nonnegative homomorphism number into any target graph with real edge weights. The Positive Graphs Conjecture offers a structural characterization: these are exactly the graphs that can be obtained by gluing together two copies of the same graph along an independent set of vertices. In this talk I will discuss our recent results on the Positive Graphs Conjecture.

Keywords: homomorphism number, quantum graph, Hilbert's seventeenth problem, positive graphs conjecture

http://dx.doi.org/10.1016/j.endm.2016.09.061

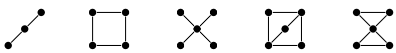
1 Positive Graphs Conjecture

Let $\hom(F, G)$ denote the number of mappings from $V(F)$ to $V(G)$ that take each edge of F into an edge of G. For instance, $\hom(C_4, G)$ is roughly the number of 4-cycles in G, but non-adjacent vertices of C_4 may be mapped into the same vertex of G.

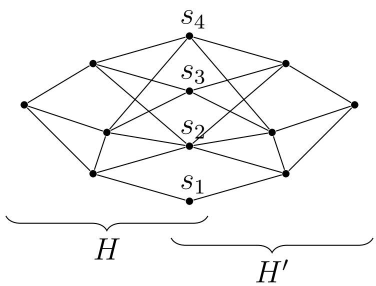
We may extend the definition of hom to allow real edge weights on the target graph G, including negative ones:

$$\hom(F, G) = \sum_{\varphi: V(F) \to V(G)} \prod_{ij \in E(F)} w_{\varphi(i)\varphi(j)}$$

We call a graph F *positive* if $\hom(F, G) \geq 0$ holds for any edge-weighted graph G. Here are some examples:



The positive graphs shown above follow a specific pattern. Suppose we have a graph H where the vertices $s_1, s_2, \ldots s_k$ form a stable set. Let H' be a disjoint copy of H and identify each s_i with s_i'. A graph F obtained this way is called *symmetric*.



It is easy to see that all symmetric graphs are positive. The Positive Graphs Conjecture [**?**] states that this implication is in fact an equivalence:

Conjecture 1.1 *G is positive $\Leftrightarrow$ G is symmetric*

The conjecture is about simple graphs, but in this talk, we'll use multigraphs to attack it. The reason for this is that there are two different notions of positivity: one for simple graphs and another for multigraphs. The one we've just defined is the natural notion in the theory of multigraphs. More on that later.

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