

On the hull number of some graph classes.¹

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Abstract

Given a graph $G = (V, E)$, the *closed interval* of a pair of vertices $u, v \in V$, denoted by $I[u, v]$, is the set of vertices that belongs to some shortest (u, v) -path. For a given $S \subseteq V$, let $I[S] = \bigcup_{u,v \in S} I[u, v]$. We say that $S \subseteq V$ is a *convex set* if $I[S] = S$.

The *convex hull* $I_h[S]$ of a subset $S \subseteq V$ is the smallest convex set that contains S . We say that S is a *hull set* if $I_h[S] = V$. The cardinality of a minimum hull set of G is the *hull number* of G , denoted by $hn(G)$.

We show that deciding if $hn(G) \leq k$ is an NP-complete problem, even if G is bipartite. We also prove that $hn(G)$ can be computed in polynomial time for cactus and P_4 -sparse graphs.

Keywords: graph convexity, hull number, bipartite graph, cactus graph, P_4 -sparse graph.

1 Introduction

In this paper, we adopt the graph terminology defined in [2]. All the graphs studied in this work are considered to be simple and undirected.

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Given a graph $G = (V, E)$, the *closed interval* of a pair of vertices $u, v \in V$, denoted by $I[u, v]$, is the set of vertices that belong to some shortest (u, v) -path. If $S \subseteq V$, let $I[S] = \bigcup_{u,v \in S} I[u, v]$. We say that $S \subseteq V$ is a *convex set* if $I[S] = S$. Given a subset $S \subseteq V$, the *convex hull* $I_h[S]$ of S is the smallest convex set that contains S . We say that S is a *hull set* if $I_h[S] = V$. The cardinality of a minimum hull set of G is the *hull number* of G , denoted by $hn(G)$. The HULL NUMBER problem is to determine whether $hn(G) \leq k$, for a given graph G and an integer k . This problem is NP-complete [5].

The HULL NUMBER problem was defined in [7]. In [8], results of abstract convexity are shown to be valid for graph convexities. An oriented version of the HULL NUMBER problem is studied in [4]. The hull number of a cartesian product of two connected graphs is characterized in [3]. Dourado et al. [6] show some bounds for the hull number of triangle-free graphs. In [5], it is also proved that the hull number of unit interval graphs, cographs and split graphs can be computed in polynomial time.

In Section 3, we answer an open question of Dourado et al. [5] by showing that it is NP-hard to compute the hull number of a given bipartite graph. We then present polynomial time algorithms to compute the hull number of cacti, in Section 4, and of P_4 -sparse graphs in Section 5. This last result extends the previous known result for cographs [5]. Finally, we present some concluding remarks.

2 Preliminaries

In order to present our main results, it is necessary to first point out some observations on hull sets.

Lemma 2.1 ([5]) *Every simplicial vertex of a graph G belongs to any hull set of G .*

Lemma 2.2 ([5]) *If a graph G is not complete, then every universal vertex u of G does not belong to any minimum hull set of G .*

Lemma 2.3 ([5]) *Let G be a graph and H be an isometric subgraph of G . Then for every hull set S of H it holds that $V(H) \subseteq I_h(S)_G$.*

Lemma 2.4 ([5]) *Let G be a graph and S a proper and non-empty subset of $V(G)$. If $V(G) \setminus S$ is convex, then every hull set of G contains at least one vertex of S .*

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