Available online at www.sciencedirect.com
SciVerse ScienceDirect
Electronic Notes in Discrete Mathematics 38 (2011) 239–244
Electronic Notes in DISCRETE MATHEMATICS
www.elsevier.com/locate/endm

Rainbow Connection Number and Connected Dominating Sets

L. Sunil Chandran [1] Anita Das [1] Deepak Rajendraprasad [1]

Department of Computer Science and Automation, Indian Institute of Science, Bangalore-560012, India

Nithin M. Varma [2]

Department of Computer Science and Engineering,National Institute of Technology, Calicut - 673 601, India

Abstract

Rainbow connection number, $rc(G)$, of a connected graph G is the minimum number of colours needed to colour the edges of G, so that every pair of vertices is connected by at least one path in which no two edges are coloured the same. In this paper we show that for every connected graph G, with minimum degree at least 2, $rc(G) \leq \gamma_c(G) + 2$, where $\gamma_c(G)$ is the connected domination number of G. Bounds of the form $diameter(G) \leq rc(G) \leq diameter(G) + c$, $1 \leq c \leq 4$, for many special graph classes follow as easy corollaries from this result. We also show that every bridge-less chordal graph G has $rc(G) \leq 3 \cdot radius(G)$. An extension of this idea to two-step dominating sets is used to show that for every connected graph on n vertices with minimum degree δ, $rc(G) \leq 3n/(\delta + 1) + 3$. This solves an open problem from Schiermeyer, 2009 [6], improving the previously best known bound of $20n/\delta$ from Krivelevich and Yuster, 2010 [5]. Moreover, this bound is seen to be tight up to additive factors by a construction mentioned in Caro et. al., 2008 [1].

Keywords: rainbow colouring, connected dominating set, minimum degree.

1571-0653/$ – see front matter
doi:10.1016/j.endm.2011.09.040

1 Introduction

Edge colouring of a graph is a function from its edge set to the set of natural numbers. A path in an edge coloured graph with no two edges sharing the same colour is called a *rainbow path*. An edge coloured graph is said to be *rainbow connected* if every pair of vertices is connected by at least one rainbow path. Such a colouring is called a *rainbow colouring* of the graph. The minimum number of colours required to rainbow colour a connected graph is called its *rainbow connection number*, denoted by $rc(G)$.

The concept of rainbow colouring was introduced in [3]. It was shown in [2] that computing the rainbow connection number of an arbitrary graph is NP-Hard.

In the search towards good upper bounds for rainbow connection number, an idea that turned out to be successful more than once is a "strengthened" notion of connected k-step dominating set: a strengthening so that a rainbow colouring of the induced graph on such a set can be extended to the whole graph using a constant number of additional colours. Theorem 1.4 in [1] was proved using a strengthened connected 1-step dominating set and Theorem 1.1 in [5] was proved using a strengthened connected 2-step dominating set. A closer examination revealed to us that the additional requirements imposed on the connected dominating sets in both those cases were far more restrictive than what was essential. This led us to the investigation of what is the weakest possible strengthening of a connected dominating set which can achieve the same. Since every edge incident on a pendant vertex will need a different colour, it is easy to see that such a dominating set should necessarily include all the pendant vertices in the graph. Quite surprisingly, it turns out that this obvious necessary condition is also sufficient! (Theorem 2.1 in Section 2). For rainbow connection number of many special graph classes, the above result gives tight upper bounds which were otherwise difficult to obtain (Theorem 2.3 in Section 2). The farthest we could get with the idea was a curious theorem about chordal graphs (Theorem 2.3(vii) in Section 2).

A similar inquiry for the weakest strengthening a connected two-step dominating set (Theorem 2.5 in Section 2) led us to the solution of an important open problem in this area regarding the optimal upper bound of rainbow connection number in terms of minimum degree. (See Theorem 2.8 in Section 2 and the remarks therein). As an intermediate step in solving the above problem, we also discovered a tight upper bound on the size of a minimum

[1] Email: {sunil, anita, deepakr}@csa.iisc.ernet.in
[2] Email: nithvarma@gmail.com