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Rainbow k-connection in Dense Graphs (Extended Abstract)

Shinya Fujita [1,2]

Department of Mathematics
Gunma National College of Technology
580 Toriba, Maebashi 371-8530, Japan

Henry Liu [3,4]

Departamento de Matemática
Faculdade de Ciências e Tecnologia
Universidade Nova de Lisboa
Quinta da Torre, 2829-516 Caparica, Portugal

Colton Magnant [5]

Department of Mathematical Sciences
Georgia Southern University
65 Georgia Ave, Statesboro, GA 30460 USA

Abstract

An edge-colouring of a graph G is *rainbow k-connected* if, for any two vertices of G, there are k internally vertex-disjoint paths joining them, each of which is *rainbow* (i.e., all edges of each path have distinct colours). The minimum number of colours for which there exists a rainbow k-connected colouring for G is the *rainbow k-connection number* of G, and is denoted by $\mathrm{rc}_k(G)$. The function $\mathrm{rc}_k(G)$ was

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introduced by Chartrand et al. in 2008, and has since attracted considerable interest. In this note, we shall consider the function $\mathrm{rc}_k(G)$ for complete bipartite and multipartite graphs, highly connected graphs, and random graphs.

Keywords: rainbow colouring, k-connected, random graph

1 Introduction

Connectivity is one of the most important and fundamental graph-theoretic concepts, in both the combinatorial setting and the algorithmic setting. The connectivity concept and its variants, such as k-connectivity, requirement of Hamiltonicity, requirement of bounded diameter, and so on, have been intensely studied. In 2008, Chartrand, Johns, McKeon and Zhang [4,5] introduced the concept of *rainbow k-connectivity*, as follows. An edge-colouring of a graph G is *rainbow k-connected* if, for any two vertices of G, there are k internally vertex-disjoint *rainbow paths* joining them (i.e., the edges of each path have distinct colours). The minimum number of colours for which there exists a rainbow k-connected colouring for G is the *rainbow k-connection number* of G, and is denoted by $\mathrm{rc}_k(G)$. We write $\mathrm{rc}(G) = \mathrm{rc}_1(G)$. Note that, by Menger's theorem, any two vertices of a graph G have k internally vertex-disjoint paths joining them if and only if G is k-connected. Hence, the function $\mathrm{rc}_k(G)$ will only be defined for k-connected graphs G.

In their original papers, Chartrand et al. [4,5] studied $\mathrm{rc}_k(G)$ for many basic families of graphs, notably in the cases when G is complete, and complete bipartite and multipartite. They also introduced the *strong rainbow connection number* $\mathrm{src}(G)$, and considered some relationships between $\mathrm{rc}(G)$ and $\mathrm{src}(G)$. Since then, the function $\mathrm{rc}_k(G)$ has attracted considerable interest. Among the concepts considered are minimum degree conditions, higher connectivity, random graphs, and the time complexity of determining $\mathrm{rc}_k(G)$, and further analogous functions were also introduced, including the *rainbow vertex connection number* $\mathrm{rvc}(G)$ and the *k-rainbow index* $\mathrm{rx}_k(G)$ (Caro et al. [2]; Krivelevich and Yuster [9]; Chartrand et al. [6], among others). Very

[1] Email: fujita@nat.gunma-ct.ac.jp

[2] Research supported by the Japan Society for the Promotion of Science for Young Scientists

[3] Email: h.liu@fct.unl.pt

[4] Research partially supported by Financiamento Base 2008 ISFL-1-297 from FCT/MCTES/PT

[5] Email: cmagnant@georgiasouthern.edu