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A min-max theorem for LIFO-search

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Abstract

We introduce a variant of the classic node search game called LIFO-search where searchers are assigned different numbers. The additional rule is that a searcher can be removed only if no searchers of lower rank are in the graph at that moment. We introduce the notion of shelters in graphs and we prove a min-max theorem implying their equivalence with the tree-depth parameter. As shelters provide escape strategies for the fugitive, this implies that the LIFO-search game is monotone and that the LIFO-search parameter is equivalent with the one of tree-depth.

Keywords: Graph Searching, Tree-depth, Obstructions, Width Parameters

1 Introduction

Node searching is a game introduced by Kirousis and Papadimitriou as a variant of the classic searching game of Parsons [6]. It was defined in terms of a graph representing a system of tunnels, a set of searchers, and an agile and omniscient fugitive with unbounded speed that is hidden on the vertices of the graph. The searcher's strategy may apply two types of moves in each step:

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either placement of a searcher on a vertex or removal of a searcher from a vertex. The fugitive is agile, i.e., may move at any moment from one vertex to an other by traversing unguarded paths, and is captured when a searcher is placed on the vertex he/she currently occupies and he/she does not have any unguarded neighboring vertex to move. Also the fugitive is invisible in the sense that the searchers strategy is given in advance and cannot take into account the location of the fugitive at any step of the search. The objective of the searchers is to capture the fugitive. Any strategy that can guarantee this is called a *capturing strategy* and its *cost* is the maximum number of searchers at any step of the search. The *contaminated positions*, at each step of the search, are all potential positions of the fugitive at that moment. A strategy that guaranties a monotone decrease of the contaminated positions is called monotone and a classic question in graph searching, is whether a search variant is *monotone*, i.e., for every capturing node-search strategy, there exists a monotone one with the same cost. The first monotonicity proof appeared in [7] while its adaptation for the node search number was given in [6]. As a consequence of its monotonocity and the result in [5], the class of graphs that have a capturing strategy of cost $k + 1$ is identical to the class of graphs with pathwidth at most k. This result revealed an interesting connection between graph searching and width-parameters. Later, the same type of equivalence was proven for other width parameters such as the treewidth [2, 11], the cutwidth [3], and the infinite degeneracy [10].

In this paper we define a variant of node searching that is equivalent to the tree-depth parameter. Tree-depth is also known as the vertex ranking problem [1], the ordered colouring problem [4], or the the minimum-height of an elimination tree of a graph [8] and has received much attention, mostly because of the theory of graph classes of bounded expansion, developed by Nešetřil and Ossona de Mendez in [9].

We set up *LIFO-search* as a variant of the classic node search game where searchers are assigned different numbers. The only additional rule is that a searcher can be removed only if no searchers of lower rank are in the graph at that moment. We call this rule the *Last In First Out* Rule to denote that, during the search, the searcher that is being removed first from the graph is the one that was placed last on it. The main result of this paper is that the class of graphs that have a capturing LIFO-search strategy of cost k is identical to the class of graphs with tree-depth at most k. To prove this we introduce the notion of a *shelter* of a graph as a max-min equivalent to tree-depth. Our proof uses the fact that shelters can serve as certificates of escape strategies for the fugitive against LIFO-search strategies.