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$SH(3)$-Gordian distances between knots with up to seven crossings

Taizo Kanenobu [a,*,1], Hiromasa Moriuchi [b]

[a] *Department of Mathematics, Osaka City University, Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan*
[b] *Faculty of Medicine, Kinki University, Onohigashi, Osakasayama, Osaka 589-8511, Japan*

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The $SH(3)$-move is an unknotting operation on oriented knots, and the $SH(3)$-Gordian distance of two knots is the minimum number of $SH(3)$-moves needed to transform one into the other, which is half of the coherent band-Gordian distance. We give a table of $SH(3)$-Gordian distances between knots with up to seven crossings.

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1. Introduction

An $SH(3)$-*move* is a local change for an oriented knot diagram involving three strands as shown in Fig. 1, which has been defined by Hoste, Nakanishi and Taniyama [7] in a more general form. Since the $SH(3)$-move is an unknotting operation, that is, any knot can be deformed into a trivial knot by a sequence of $SH(3)$-moves, we may define the $SH(3)$-Gordian distance between two knots and the $SH(3)$-unknotting number for a knot. The main result of this paper is Table 1, which lists the $SH(3)$-Gordian distances between knots with up to seven crossings; 1-2 means that the distance is either 1 or 2. We have another local move for an oriented link diagram as shown in Fig. 2 called a *coherent band surgery*, where the number

* Corresponding author.
E-mail addresses: kanenobu@sci.osaka-cu.ac.jp (T. Kanenobu), moriuchi@med.kindai.ac.jp (H. Moriuchi).
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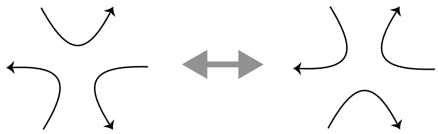

Fig. 1. An $SH(3)$-move.

Table 1
$SH(3)$-Gordian distances between knots with up to seven crossings.

	3_1	4_1	5_1	5_2	6_1	6_2	6_3	$3_1\#3_1$	$3_1!\#3_1$	7_1	7_2	7_3	7_4	7_5	7_6	7_7	$3_1\#4_1$
U	1	1	2	1	1	1	1	2	1	3	1	2	1	2	1	1	1
3_1	0	1	1	1	2	1	1	1	1	2	1	3	2	1	1	1	1
$3_1!$	2	1	3	2	1	2	1	3	1	4	2	1	1	3	2	1	2
4_1		0	2	1	1	1	1	2	2	3	1	2	1-2	2	1	1	1
5_1			0	1	2	1	2	1	2	1	1	4	3	1	1	2	1
$5_1!$			4	3	2	3	2	4	2	5	3	1	1	4	3	2	3
5_2				0	1	1	1	1	2	2	1	3	2	1	1	1	1
$5_2!$				2	1	2	1	3	2	4	2	1	1	3	2	1-2	2
6_1					0	1	1	2	1	3	1-2	2	2	2	1	2	1
$6_1!$					1	1	1	2	1	3	1	2	1	2	1	1	2
6_2						0	1	2	1	2	1	3	2	1-2	1	1	1
$6_2!$						2	1	3	1	4	2	1	1-2	3	2	1	2
6_3							0	2	1	3	1	2	1-2	2	1	1	1
$3_1\#3_1$								0	2	1	2	4	3	1	1	2	1
$3_1!\#3_1!$								4	2	5	3	2	1	4	3	2	3
$3_1!\#3_1$									0	3	1-2	2	1-2	2	2	1	1
7_1										0	2	5	4	1	2	3	2
$7_1!$										6	4	1	2	5	4	3	4
7_2											0	3	2	1	1	1	1-2
$7_2!$											2	1	1	3	2	1-2	2
7_3												0	1	4	3	2	3
$7_3!$												4	3	1	1	2	1-2
7_4													0	3	2	1-2	2
$7_4!$													2	1	1	2	1
7_5														0	1	2	1
$7_5!$														4	3	2	3
7_6															0	1	1
$7_6!$															2	1	2
7_7																0	2
$7_7!$																2	1
$3_1!\#4_1$																	2

of the components of a link changes by one. Since we may regard smoothing a crossing as a coherent band surgery, any knot can be deformed into a trivial link by a sequence of coherent band surgeries. So, we may define the coherent band-Gordian distance between two oriented links. For an oriented link L, we define the coherent band-unknotting number of L to be the coherent band-Gordian distance from L to the trivial knot. Then the $SH(3)$-Gordian distance is half of the coherent band-Gordian distance between two knots (Proposition 2.1). The first author [14] has given the table of the $SH(3)$-unknotting numbers for knots with up to nine crossings, and Buck and Ishihara [8] have given tables of the coherent band-Gordian distances between links with up to six crossings, which imply the table of the $SH(3)$-Gordian distances between knots with up to six crossings. In this paper, we extend these tables. In order to give a lower bound of the $SH(3)$-Gordian distance, the signature is the most useful tool (Proposition 2.4). Besides this, we can make use of some special values of the Jones, Q, and HOMFLYPT polynomials, which are related to the homology group of the branched cyclic covering space along the link; see [14–16]. However, for the knots with up to seven crossings, we can only use the Jones polynomial (Corollaries 3.2 and 3.6); in particular, Corollary 3.6 is implied by Theorem 3.5, which is a new criterion for links with coherent band-Gordian distance two.

For an upper bound of the $SH(3)$-Gordian distance, we have a usual Gordian distance between knots (Proposition 2.2). So, we can make use of the table of Darcy [3,4]. Furthermore, we give a table of knots and links related by a single coherent band surgery with up to seven crossings (Table 4), which allows us to give pairs of knots with $SH(3)$-Gordian distance one, since if two knots have a common link with

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