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An unknotting sequence for torus knots ☆

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ABSTRACT

In this paper, the authors give an unknotting sequence for torus knots and also determine the unknotting numbers of ${}_n14_{17191}$, ${}_n14_{14274}$, ${}_n14_{18351}$, ${}_n14_{24498}$ and some other knots from the knot table of Hoste–Thistlethwite.
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1. Introduction

An unknotting sequence for a knot or a link K is a finite sequence of knots or links

$$K = K_n, K_{n-1}, K_{n-2}, \cdots, K_1, K_0 = \text{trivial link},$$

such that:

1. The unknotting number of K_i is i, i.e. $u(K_i) = i$, $0 \leq i \leq n$, and
2. two succeeding knots or links of the sequence are related by one crossing change.

Even though every knot in S^3 can be unknotted by a finite sequence of crossing changes, it is interesting to see that every knot of unknotting number at least two can be unknotted via infinitely many different

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knots of unknotting number one [3]. Sebastian, in his paper [2], showed that the unknotting number of a quasipositive knot is equal to its genus if and only if it lies in an unknotting sequence of some torus knot. However, many non-quasipositive knots also exist in unknotting sequences of torus knots. For example, the non-quasipositive knots 8_2 and 8_7 exist in unknotting sequences 5_1, 8_2, 0_1 and 5_1, 8_7, 0_1 respectively.

In [1], the authors presented a new approach to unknot torus knots and extended the same to torus links. In particular, the authors introduced unknotting crossing data and minimal unknotting crossing data in [1]. This minimal unknotting crossing data helps in selecting a pattern of crossings from a toric braid representation of torus knots, such that switching of all crossings at this selected crossing data results a braid whose closure is isotopically equivalent to the trivial knot. Based on this selection of pattern of crossings, authors determine the exact braid representation of all intermediate braids, whose closures give an unknotting sequence for torus knots.

In Section 2, the authors give an unknotting sequence for torus knots. In Section 3, the unknotting numbers of ${}_n13_{604}$, ${}_n14_{17191}$, ${}_n14_{14274}$, ${}_n14_{18351}$, ${}_n14_{24498}$ and some other knots is obtained by showing that each of these knots lies in some unknotting sequence of torus knots.

In Subsection 3.1, a sharp upper bound for the unknotting number of two special classes of knots has been discussed.

2. An unknotting sequence of torus knots

The unknotting number of torus knots is well known [4,5]. Here we give an unknotting sequence for torus knots. Throughout this paper, we use term torus knots for both torus knots and torus links. We consider torus knots as the closure of $(\sigma_1\sigma_2\cdots\sigma_{p-1})^q$ and denote as $K(p,q) = cl(\sigma_1\sigma_2\cdots\sigma_{p-1})^q$. We denote $1+2+\cdots+n$ as $\sum n$ and use two braid types

$$\eta_l = \begin{cases} \sigma_{k+2-l}\sigma_{k+3-l}\cdots\sigma_{p-l} & \text{if } 1 \leq l < j \\ \sigma_{k+3-l}\sigma_{k+4-l}\cdots\sigma_{p-l} & \text{if } j < l < k+2 \end{cases} \text{ and } \beta_j = \sigma_{k+3-j}\sigma_{k+4-j}\cdots\sigma_{p-1},$$

which depends on j and k (we will define j and k whenever we use η_l and β_j).

To find an unknotting sequence of torus knots, we divide all torus knots in two classes

1. when $q \equiv 0$ or $\pm 1 \pmod p$
2. otherwise

Remark 1. Observe that for a sequence of knots $K_n, K_{n-1}, \cdots K_m, \cdots, K_0$, where K_0 is trivial knot and K_i can be obtained from K_{i+1} by one crossing change for any i, if $u(K_m) = m$ for some m then $K_m, K_{m-1}, \cdots, K_0$ is an unknotting sequence for K_m.

Theorem 2.1. *Let $q = p$ or $p \pm 1$, $n = u(K(p,q))$ and for any $i \leq n$*

$$K_{n-i} = cl(\eta_1\eta_2\cdots\eta_{j-1}\beta_j\eta_{j+1}\cdots\eta_{k+2}(\sigma_1\sigma_2\cdots\sigma_{p-1})^{q-(k+2)})$$

where,

$$k = \sup\{n \in \mathbb{Z}^+ : \sum n < i\},$$

$$j = k + 2 - (i - \sum k),$$

then

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