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Canonical decompositions of hyperbolic fibered two-bridge link complements

Naoki Sakata

Department of Mathematics, Graduate School of Science, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima, 739-8526, Japan

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ABSTRACT

We prove that the canonical decompositions of hyperbolic fibered two-bridge link complements are layered triangulations, by showing that A'Campo's criterion for detecting fiberedness works for all hyperbolic fibered two-bridge links.

1. Introduction

By the work of Epstein and Penner [6] (see also [14]), each finite-volume cusped hyperbolic manifold admits a *canonical decomposition* into ideal polyhedra. In his famous unfinished work [10], Jorgensen described the canonical decompositions of once-punctured torus bundles over S^1 (cf. [3,8]). They are constructed by using the fiber structures, and in fact they are "layered" triangulations with respect to the fiber structure (see Section 2 for the definition). It is natural to ask if a similar result holds for the canonical decompositions of more general hyperbolic punctured surface bundles over S^1. The main purpose of this paper is to give a positive answer to this question for hyperbolic fibered two-bridge link complements.

Main Theorem. *The canonical decompositions of hyperbolic fibered two-bridge link complements are layered triangulations with respect to the fiber structure.*

E-mail address: sakata202988e-05@hiroshima-u.ac.jp.

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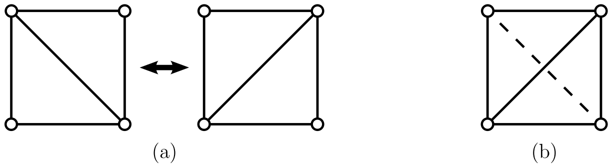

Fig. 1. (a) A Whitehead move. (b) A Whitehead move realized by attaching an ideal tetrahedron.

To prove this result, we use Norbert A'Campo's criterion for detecting fiberedness (Theorem 2.2), and we verify that his criterion works for all hyperbolic two-bridge links (Theorem 2.6).

2. A'Campo's criterion for detecting fiberedness

An ideal triangulation T of a punctured surface Σ is a decomposition along ideal edges into ideal triangles. A *Whitehead move of* T takes an ideal edge in T which is adjacent to two distinct ideal triangles, removes it, and replaces it with the other diagonal of the quadrilateral (see Fig. 1(a)). Then we have a new ideal triangulation, T', of Σ. Note that the Whitehead move $T \to T'$ is realized by attaching an ideal tetrahedron to T along a pair of ideal triangles of T sharing an ideal edge (see Fig. 1(b)).

For a homeomorphism $h : \Sigma \to \Sigma$, let $\mathcal{T}(h) = \Sigma \times [0,1]/\{(x,0) \sim (h(x),1)\}$ be the mapping torus of h. Consider a sequence of Whitehead moves $T = T_0 \to T_1 \to \cdots \to T_m = h(T)$ transforming T into $h(T)$. Then, as described in [2, Section 4], if h is pseudo-Anosov, we obtain an ideal triangulation of $\mathcal{T}(h)$ consisting of the ideal tetrahedra corresponding to the Whitehead moves. This triangulation is called a *layered triangulation with respect to the fiber structure*.

Let $\mathcal{D}$ be an ideal triangulation of a cusped hyperbolic 3-manifold of finite volume M. Let $\mathcal{F}$ be the 2-cell complex dual to $\mathcal{D}$. Namely,

(1) The i-cells of $\mathcal{F}$ are dual to the ideal $(3-i)$-simplices of $\mathcal{D}$ for $0 \le i \le 2$.
(2) For an ideal triangle f of $\mathcal{D}$, the edge (1-cell) f^* of $\mathcal{F}$ dual to f joins the pair of (possibly identical) vertices (0-cells) of $\mathcal{F}$ dual to the pair of ideal tetrahedra of $\mathcal{D}$ sharing f.
(3) Let e be an ideal edge of $\mathcal{D}$ and let $f_1, \dots, f_n$ be the ideal triangles of $\mathcal{D}$ sharing the ideal edge e which are arranged around e in this cyclic order. Then the boundary of the face (2-cell) e^* dual to e consists of the edges (1-cells) $f_1^*, \dots, f_n^*$ of $\mathcal{F}$.

It should be noted that each edge of $\mathcal{F}$ is shared by (the germs of) three faces of $\mathcal{F}$ and that each vertex of $\mathcal{F}$ is shared by (the germs of) four edges and six faces of $\mathcal{F}$.

Let $\mathrm{C}_*(\mathcal{F};R) = \{(\mathrm{C}_i(\mathcal{F};R), \partial_i)\}$ and let $\mathrm{C}^*(\mathcal{F};R) = \{(\mathrm{C}^i(\mathcal{F};R), \delta^i)\}$ be the chain complex and the cochain complex of $\mathcal{F}$, respectively, with coefficients in a commutative ring R. If R is the real number field $\mathbb{R}$, we drop the symbol R.

Definition 2.1. Let ω be an element of $\mathrm{C}^1(\mathcal{F})$.

(1) We say that ω is *balanced at a vertex* v of $\mathcal{F}$ if $\omega(e)$ is positive for two oriented edges e with initial point v and $\omega(e)$ is negative for two oriented edges e with initial point v.
(2) We say that ω is *balanced* if it is balanced at every vertex of $\mathcal{F}$.

By a *fibration* of M, we mean a bundle map $p : M \to S^1$ such that $p^*(1) \in \mathrm{H}^1(\mathcal{F};\mathbb{Z})$ is primitive, where 1 denotes a generator of $\mathrm{H}^1(S^1;\mathbb{Z})$. Note that, an oriented link is a fibered link if and only if the link exterior admits a fibration each of whose fiber is a Seifert surface of the oriented link.