Topology and its Applications 196 (2015) 860–867

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

The reductivity of spherical curves

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ARTICLE INFO

Article history:
Received 6 September 2013
Received in revised form 15 January 2014
Accepted 15 January 2014
Available online 27 May 2015

MSC:
57M25

Keywords:
Half-twisted splice
Knot projection
Reduced knot projection
Reductivity
Spherical curve

ABSTRACT

We show that we can obtain a reducible spherical curve from any non-trivial spherical curve by four or less inverse-half-twisted splices, i.e., the reductivity, which represents how reduced a spherical curve is, is four or less. We also discuss unavoidable sets of tangles for spherical curves.

1. Introduction

A *spherical curve* is a smooth immersion of the circle into the sphere where the self-intersections are transverse and double points (we call the double point *crossing*). In this paper we assume every spherical curve is oriented, and has at least one crossing. We represent, if necessary, the orientation of a spherical curve by an arrow as depicted in the left-hand side of Fig. 1. A spherical curve P is *reducible* and has a *reducible crossing* p if P has a crossing p as shown in Fig. 2, where T and T' imply parts of the spherical curve. P is *reduced* if P is not reducible such as the spherical curves in Fig. 1. Note that around a reducible (resp. non-reducible) crossing, there are exactly three (resp. four) disjoint regions, where a *region* of a spherical curve is a part of the sphere divided by the spherical curve.

A *half-twisted splice* is the local transformation on spherical curves as depicted in Fig. 3 [2,4]. Then the inverse is the transformation depicted in Fig. 4. In this paper we call the inverse of the half-twisted splice *inverse-half-twisted splice*, and denote by I. We remark that the half-twisted splice and the inverse-half-twisted splice do not preserve the orientation of spherical curves. Then we give an orientation again to the

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http://dx.doi.org/10.1016/j.topol.2015.05.055
0166-8641/

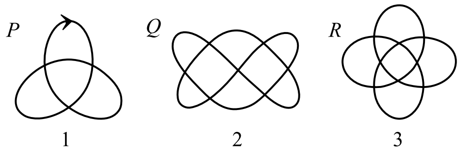

Fig. 1. Spherical curves.

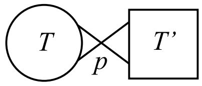

Fig. 2. Reducible spherical curve.

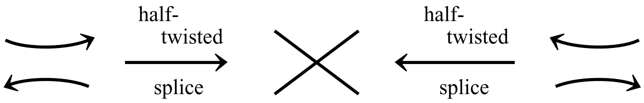

Fig. 3. Half-twisted splice.

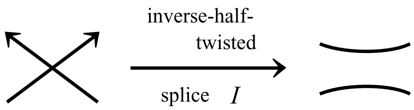

Fig. 4. Inverse-half-twisted splice.

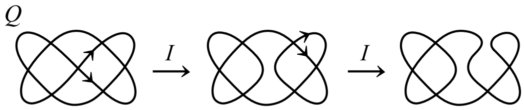

Fig. 5. The reductivity of Q is 2.

spherical curve we obtain. We also remark that the half-twisted splice and the inverse-half-twisted splice do not depend on the orientations of spherical curves, but depend only on the relative orientations. Now we define the reductivity. The *reductivity* $r(P)$ of a spherical curve P is the minimal number of I which are needed to obtain a reducible spherical curve from P. For example, a reducible spherical curve has the reductivity 0, and the spherical curves P, Q and R in Fig. 1 have the reductivity 1, 2 and 3, respectively (see Fig. 5 for Q; note that we cannot obtain a reducible curve by any single I from Q). In this paper, we show the following theorem:

Theorem 1.1. *Every spherical curve has the reductivity four or less.*

This implies that we can obtain a reducible spherical curve from any spherical curve by four or less I. We have the following question.

Question 1.2. *Is it true that every spherical curve has the reductivity three or less?*

In other words, is it true that there are no spherical curve with reductivity four? The rest of this paper is organized as follows: In Section 2, we discuss the properties of reductivity by considering chord diagrams, and prove Theorem 1.1. In Section 3, we study the unavoidable sets of tangles for spherical curves as an approach to Question 1.2.