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Universal sequences of spatial graphs

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ABSTRACT

An increasing sequence of integers is said to be universal for links if every link has a projection to the sphere such that the number of edges of each complementary face of the projection image comes from the given sequence. In this paper, we extend the concept of universal sequences to spatial graphs and prove that the sequence $(2, 3, 4, 5)$ is universal for spatial graphs without vertices of odd degree.

1. Introduction

We assume that a graph does not have any vertices of degrees 0 and 1. A spatial graph is the image of an embedding $f : \Gamma \to \mathbb{S}^3$ of a finite graph Γ. In particular, $f(\Gamma)$ is called a *knot* if Γ is homeomorphic to a circle and $f(\Gamma)$ is called an *n-component link* if Γ is homeomorphic to disjoint n circles. Let D be a diagram of spatial graph G on the 2-sphere $\mathbb{S}^2$ and $|D|$ be the graph obtained from D by replacing each crossing with a vertex. We call the vertices of $|D|$ corresponding to the vertices of G *original vertices* and the vertices of $|D|$ corresponding to the crossings of D *crossing vertices*. We call each connected component of $\mathbb{S}^2 - |D|$ a *face* of D. We remark that if D is a link diagram, then $|D|$ is a four-valent graph on $\mathbb{S}^2$. A face of D is said to be an *n-gon* or an *n-sided face* if the face has n crossings on its boundary. A crossing c in a spatial graph diagram D is called a *reducible* crossing if D can be represented as in Fig. 1. A diagram D is called *reducible* if it has a reducible crossing, otherwise it is called *reduced*. Let D be a reducible diagram with n crossings. As in the figure, the diagram D' with $n - 1$ crossings can be obtained from D where c is a reducible crossing. These two diagrams D, D' represent the same spatial graph.

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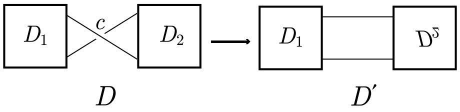

Fig. 1. A reducible crossing.

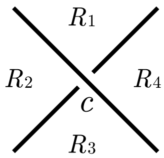

Fig. 2. Faces around a crossing.

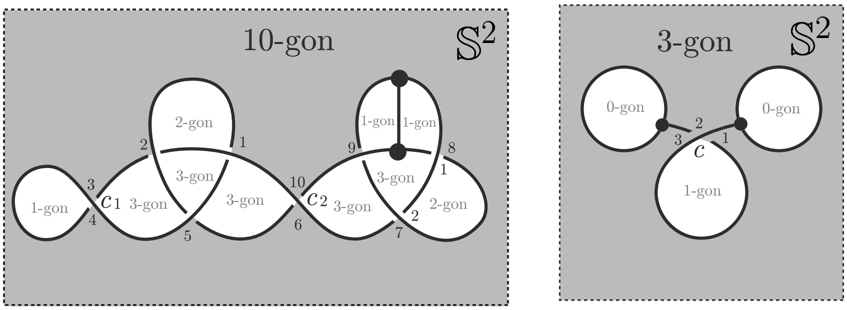

Fig. 3. Diagrams which have a reducible crossing.

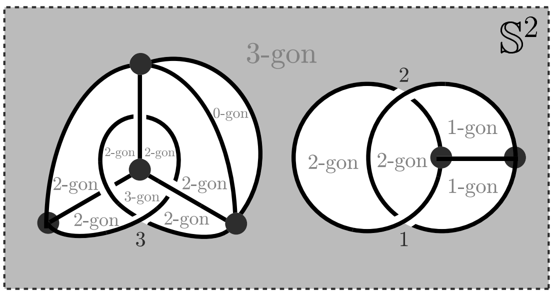

Fig. 4. A disconnected diagram.

Remark 1.1.

(i) If a crossing c in Fig. 2 is a reducible crossing, the faces R_1, R_2, R_3 and R_4 around c satisfy $R_1 = R_3$ or $R_2 = R_4$.

(ii) Let F be a face of a diagram and c a reducible crossing on the boundary of F. If F contains two of the four faces R_1, R_2, R_3 and R_4 around c, we count c twice on the boundary of F. For example see the diagram on the left-hand side in Fig. 3. The shaded face is a 10-gon. If F contains three of the four faces R_1, R_2, R_3 and R_4 around c, we count c three times on the boundary of F. For example see the diagram on the right-hand side in Fig. 3. The shaded face is a 3-gon.

(iii) The shaded face in Fig. 4 has two boundary components. We regard the face as a 3-gon, since the shaded face has three crossings on its boundary.