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On the $ER(2)$-cohomology of some odd-dimensional projective spaces

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ABSTRACT

Kitchloo and Wilson have used the homotopy fixed points spectrum $ER(2)$ of the classical complex-oriented Johnson–Wilson spectrum $E(2)$ to deduce certain non-immersion results for real projective spaces. $ER(n)$ is a $2^{n+2}(2^n-1)$-periodic spectrum. The key result to use is the existence of a stable cofibration $\Sigma^{\lambda(n)}ER(n) \to ER(n) \to E(n)$ connecting the real Johnson–Wilson spectrum with the classical one. The value of $\lambda(n)$ is $2^{2n+1}-2^{n+2}+1$. We extend Kitchloo–Wilson's results on non-immersions of real projective spaces by computing the second real Johnson–Wilson cohomology $ER(2)$ of the odd-dimensional real projective spaces RP^{16K+9}. This enables us to solve certain non-immersion problems of projective spaces using obstructions in $ER(2)$-cohomology.

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1. Introduction

The spectrum MU of complex cobordism comes naturally equipped with an action of $\mathbb{Z}/2$ by complex conjugation. Hu and Kriz in [4] have used this action to construct genuine $\mathbb{Z}/2$ equivariant spectra $E\mathbb{R}(n)$ from the complex-oriented spectra $E(n)$. Kitchloo and Wilson in [6] have used the homotopy fixed point spectrum of this to solve certain non-immersion problems of real projective spaces. The homotopy fixed point spectrum $ER(n)$ is $2^{n+2}(2^n-1)$-periodic compared to the $2(2^n-1)$-periodic $E(n)$. The spectrum $ER(1)$ is $KO_{(2)}$ and $E(1)$ is $KU_{(2)}$.

Kitchloo and Wilson have demonstrated the existence of a stable cofibration connecting $E(n)$ and $ER(n)$,

$$\Sigma^{\lambda(n)}ER(n) \xrightarrow{x} ER(n) \to E(n) \tag{1}$$

where $\lambda(n) = 2^{2n+1}-2^{n+2}+1$. This leads to a Bockstein spectral sequence for x-torsion. It is known that $x^{2^{n+1}-1}=0$ so there can be only $2^{n+1}-1$ differentials. For the case of our interest $n=2$ there are only 7 differentials.

From [5] we know that if there is an immersion of RP^b to $\mathbb{R}^c$ then there is an axial map

$$RP^b \times RP^{2^L-c-2} \to RP^{2^L-b-2} \tag{2}$$

For $b=2n$ and $c=2k$ Don Davis shows in [2] that there is no such map when $n=m+\alpha(m)-1$ and $k=2m-\alpha(m)$, where $\alpha(m)$ is the number of ones in the binary expression of m by finding an obstruction to James's map (2) in $E(2)$-cohomology. Kitchloo and Wilson get new non-immersion results by computing obstructions in $ER(2)$-cohomology. In this paper we extend Kitchloo–Wilson's results by computing the $ER(2)$-cohomology of the odd projective space RP^{16K+9}. This will give us newer non-immersion results. The main results are the following.

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Theorem 1.1. *A 2-adic basis of $ER(2)^{8*}(RP^{16K+9}, *)$ is given by the elements*

$$\begin{aligned}
&\alpha^k u^j \quad (k \geqslant 0,\ 1 \leqslant j \leqslant 8K+4)\\
&v_2^4\alpha^k u^j \quad (k \geqslant 1,\ 1 \leqslant j \leqslant 8K+4)\\
&v_2^4 u^j \quad (4 \leqslant j \leqslant 8K+4)\\
&x\alpha^k i_{16K+9}, \quad xv_2^4\alpha^k i_{16K+9} \quad (k \geqslant 0)
\end{aligned}$$

Theorem 1.2. *Let $\alpha(m)$ be the number of ones in the binary expansion of m. If $(m, \alpha(m)) \equiv (6, 2)$ or $(1, 0) \bmod 8$, $RP^{2(m+\alpha(m)-1)}$ does not immerse in $\mathbb{R}^{2(2m-\alpha(m))+1}$.*

This shall give us new non-immersions that are often new and different from those of [6] and [7]. Using Davis's table [1] the first new result is $RP^{2^{13}-2}$ does not immerse in $\mathbb{R}^{2^{14}-59}$.

2. The Bockstein spectral sequence

The results obtained in this section can be found in [6]. We reproduce it here for the convenience of the reader.

We have the stable cofibration

$$\Sigma^{\lambda(n)} ER(n) \xrightarrow{x} ER(n) \to E(n)$$

where $x \in ER(n)^{-\lambda(n)}$ and $\lambda(n) = 2^{2n+1} - 2^{n+2} + 1$. The fibration gives us a long exact sequence

$$\begin{array}{ccc}
ER(n)^*(X) & \xrightarrow{\quad x \quad} & ER(n)^*(X)\\
 & {}_{\partial}\nwarrow \quad \swarrow_{\rho} & \\
 & E(n)^*(X) &
\end{array} \tag{3}$$

where x lowers the degree by $\lambda(n)$ and ∂ raises the degree by $\lambda(n)+1$. This leads to the Bockstein spectral sequence, which will completely determine $M = ER(n)^*(X)/(x)$ as a subring of $E(n)^*(X)$. We know that $x^{2^{n+1}-1} = 0$ so there can be only $2^{n+1} - 1$ differentials.

We filter M,

$$0 = M_0 \subset M_1 \subset M_2 \subset \cdots \subset M_{2^{n+1}-1} = M$$

by submodules

$$M_r = \operatorname{Ker}\left[x^r : \frac{ER(n)^*(X)}{x} \to \frac{x^r ER(n)^*(X)}{x^{r+1}}\right]$$

so that M_r/M_{r-1} gives the x^r-torsion elements of $ER(n)^*(X)$ that are non-zero in M.

We collect the basic facts about the spectral sequence in the following theorem. $E(n)$ is a complex-oriented spectrum with a complex conjugation action. Denote this action by c.

Theorem 2.1. *([6, Theorem 4.2]) In the Bockstein spectral sequence for $ER(n)^*(X)$*

1. *The exact couple (3) gives rise to a spectral sequence, E^r, of $ER(n)^*$-modules, starting with*

$$E^1 \simeq E(n)^*(X)$$

2. $E^{2^{n+1}} = 0$.
3. $\operatorname{Im} d^r \simeq M_r/M_{r-1}$.
4. *The degree of d^r is $r\lambda(n)+1$.*
5. $d^r(ab) = d^r(a)b + c(a)d^r(b)$.
6. $d^1(z) = v_n^{-(2^n-1)}(1-c)(z)$ *where* $c(v_i) = -v_i$.
7. *If $c(z) = z$ in E^1, then $d^1(z) = 0$. If $c(z) = z$ in E^r then $d^r(z^2) = 0$.*
8. *The following are all vector spaces over $\mathbb{Z}/2$:*

$$M_j/M_i \quad (j \geqslant i > 0) \quad \text{and} \quad E^r \quad (r \geqslant 2)$$

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