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The problem of coincidence in a theory of temporal multiple recurrence

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ABSTRACT

Logical theories have been developed which have allowed temporal reasoning about eventualities (à la Galton) such as states, processes, actions, events and complex eventualities such as sequences and recurrences of other eventualities. This paper presents the problem of coincidence within the framework of a first order logical theory formalizing temporal multiple recurrence of two sequences of fixed duration eventualities and presents a solution to it.

The coincidence problem is described as: *if two complex eventualities (or eventuality sequences) consisting respectively of component eventualities* $x_0, x_1, \ldots, x_r$ *and* $y_0, y_1, \ldots, y_s$ *both recur over an interval* k *and all eventualities are of fixed durations, is there a subinterval of* k *over which the incidence* x_t *and* y_u *for* $0 \leq t \leq r$ *and* $0 \leq u \leq s$ *coincide*? The solution presented here formalizes the intuition that a solution can be found by temporal projection over a cycle of the multiple recurrence of both sequences.

1. Introduction

Over time, a substantial body of literature has built up in the area of temporal knowledge representation and reasoning. Time has been studied as time points (or instants) and time intervals or moments [2,8,10]. Each of these views as to what should constitute the basic elements in the time ontology gives rise to different fundamental problems, and each proposition usually has as its motivation the solution to some particular reasoning problem or task.

In addition to the time ontology, there is also the need to identify the propositions whose truth values must be evaluated over time. A theory evaluating these is known as a theory of temporal incidence [17]. We refer to these propositional entities as *eventualities* in this paper following Galton [7] and generalizing Vila's fluent/state dichotomy [17]. Various taxonomies of temporal propositions have emerged over the years, starting from McDermott's dichotomy of facts and events to Allen's states/events/processes trichotomy [3], to Shoham's more elaborate classification, based on temporal properties [16].

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Vila's survey [18] identified two basic tasks required to support the kind of temporal reasoning that needs to be done in medical diagnosis, planning, industrial process supervision and natural language understanding. These are *temporal consistency maintenance*, which involves maintaining consistency in a temporal data base while adding new entries, and *temporal question answering*, which involves "providing answers to queries" that require temporal knowledge.

In a sense temporal reasoning research has been addressing the problem of how to provide answers to the truth status (i.e. temporal incidence) of some propositional eventualities in the presence of certain well known temporal phenomenon such as recurrence. Koomen [10] had extended Allen's theory of action and time to accommodate the ability to reason about "recurrence". Precisely, Koomen developed a logical theory, which would enable temporal reasoners to reason about the concept of recurrence of both simple eventualities (such as events and states) and their sequences over time. Thus given that an alternating sequence of eventualities recurs over time, is it possible to find a future time interval within the interval of recurrence over which the latter of the two eventualities will be true. For example Koomen's theory enabled the reasoner to help a robot driver to reason on encountering a red light at a junction that the existence of a recurring sequence of red and green light will imply that at some point in the future the green light will come up. Thus encountering a red light at the junction is not good enough reason to abandon the driver's plan of getting to his destination. The temporal behavior of recurrent (or repetitive) eventualities, has been studied in the literature [1] treating them as ontological entities.

Koomen's notion of recurrence contrasts with some others in the literature such as Pan [13]. While the notion of recurrence by Koomen implies a periodic repetition of an event, state or some other propositional eventuality with contiguous periods that a planner may need to track, Pan uses recurrence to refer to repeated incidence of a propositional eventuality at regular intervals, such as "a meeting holding every Tuesday", as may arise in natural language discourse.

Our work here is motivated by the problem of how to enable a planner to cope with a slightly more complex scenario than Koomen's, which is reasoning about the possible incidence of the preconditions of an action in the context of regular *multiple* recurrence as opposed to a *single* one. Given that two sequences of eventualities $x_0, x_1, \ldots, x_r$ and $y_0, y_1, \ldots, y_s$ (such that any pair of eventualities from the x sequence or the y sequence are mutually exclusive) both recur over some interval, and a planner requires the coincidence of an eventuality pair x_p and y_q from the sequences, in order to carry out a certain action. Can a planner infer the existence (or otherwise) of such an interval which is a subinterval of two intervals over which each of x_p and y_q are both true. In other words can a planner predict the incidence or otherwise of such a coincidence?

Suppose for example, that in addition to the traffic light at the junction in Koomen's example, there is a gate that is shut, at the top of every hour for fifteen minutes to enable some house keeping to be done in certain parts of the road network. Then, the planner must reason about the existence of some time in the future when both the gate will be open, and there will be green light at the junction.

Another simple example of this from a factory domain is a piece of factory machinery which must work continuously for five days which must be followed by a maintenance process that must last three continuous days, but the engineer will not be available on a Wednesday. In this case a scheduler must reason about the avoidance of a situation in which the maintenance period starts or ends on a Wednesday. In both cases the scheduler needs to acquire the capability to determine the possibility of the simultaneous incidence of two different conditions from two different recurring sequences.

In the first example, the planner must determine the existence of a future time interval over which the gate is open and there is green light at the junction i.e. the preconditions for his proposed action of driving past the point, while in the second, a scheduler must hope that he can avoid a situation in which one of the three consecutive maintenance days is a Wednesday. Now we consider the second example. If the factory starts working on a Monday, then the first maintenance days will start on a Saturday and last till the following Monday; the next one will be a Sunday to Tuesday; while the third one will be a Monday to