Journal of Applied Logic 13 (2015) 557–581

Contents lists available at ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

On the complexity of bribery and manipulation in tournaments with uncertain information ☆

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ARTICLE INFO

Article history:
Available online 18 March 2015

Keywords:
Computational social choice
Bribery
Tournaments
Ranking
Uncertainty

ABSTRACT

We study the computational complexity of bribery and manipulation schemes for sports tournaments with uncertain information. We introduce a general probabilistic model for multi-round tournaments and consider several special types of tournament: challenge (or caterpillar); cup; and round robin. In some ways, tournaments are similar to the sequential pair-wise, cup and Copeland voting rules. The complexity of bribery and manipulation are well studied for elections, usually assuming deterministic information about votes and results. We assume that for tournament entrants i and j, the probability that i beats j and the costs of lowering each probability by fixed increments are known to the manipulators. We provide complexity analyses for several problems related to manipulation and bribery for the various types of tournaments. Complexities range from probabilistic log space to $\mathrm{NP}^{\mathrm{PP}}$. This shows that the introduction of uncertainty into the reasoning process drastically increases the complexity of bribery problems in some instances.

1. Introduction

Sports competitions are common forms of entertainment and recreation around the world. In most sports contests both observers and players have some notion of which competitors are favored over others. Many individuals, including some players, wager vast sums of money on the outcomes of particular games and

☆ A preliminary version of this paper appears in the proceedings of the *Twenty-Fifth International Florida Artificial Intelligence Research Society Conference (FLAIRS 2012)*, 2012 [35] and in Nicholas Mattei's Doctoral Dissertation [33]. Most of the work on this project occurred while the first author was a graduate student at the University of Kentucky.

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[1] Supported in part by NSF grants CCF-1049360, ITR-0325063, and IIS-1107011. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

[2] NICTA is funded by the Australian Government through the Department of Communications and the Australian Research Council through the ICT Centre of Excellence Program.

http://dx.doi.org/10.1016/j.jal.2015.03.004
1570-8683/

tournaments. A quick Google search reveals dozens of players, coaches, referees, and judges convicted of manipulating the outcome of sports competitions through match fixing, point shaving, and outright cheating. Additionally many websites (such as www.kenpom.com) produce and publish in-depth statistics for overall team win/loss predications, predictions for individual player stats on a per game basis, and real-time win probability graphs (which dynamically update their prediction based on in-game events). It is a world of probabilities and manipulation.

We use sports tournaments as a motivating example of domains in which bribery [14] and manipulation by coalitions [10] can undermine the integrity of competition. Using tournaments to select alternatives has received attention (and use) in a variety of fields including AI [52,56], economics [42], and operations research [50]. Tournaments and single winner elections, when the set of candidates and the set of voters are equivalent, are used in many domains including self-organization of ad hoc wireless sensor networks, where leaders are elected to delegate work or act as central routing nodes [46], and web ranking, where page rankings are sometimes determined by links from the set of pages under consideration [8]. In addition to these important applications of tournaments, there has been recent empirical research in political science and sociology revealing that, in the United States, voter preferences in political elections can be significantly affected by apparently irrelevant events, such as sports tournaments [25]. Our research sheds light on the security of tournaments to outside influence. We show that some forms of manipulation are easy and, thus, warn against using these methods for social choice without additional precautions.

The problems we consider are of an abstract nature. We define a general model of sports tournaments where we assume that we can pay an opposing team to not compete to the best of its ability. We provide a parameter to fine tune the fidelity of the model. While it may be difficult or impossible to exactly quantify the impact that say, paying a team's best player to play suboptimally, would have on the outcome probabilities of the match, there are estimations of such impacts available (e.g. http://predictionmachine.com/). Our tournament manipulation and bribery problems have a similar feel to the coalition manipulation and bribery problems often studied in social choice and voting [9,10,14]. However, as we will discuss, there are key differences that do not allow many of these results to transfer in a straightforward way.

In this paper we study general sports tournaments consisting of a series of rounds, where each round consists of a set of matches determined by the outcomes of the matches in previous rounds. Throughout the paper we will use "tournament" in its canonical sense as a sports or matchup competition and not in its strict mathematical sense as a complete directed graph [31]. We will use the equivalent and correct term *complete majority relation* when referring to the mathematical object known as a tournament. We consider several special types of sports tournaments.

General tournament: A tournament consists of a series of rounds, each round consisting of a set of matches between pairs of entrants. Which matches occur in the ith round depends on the number n of entrants and the outcomes of the matches in the preceding rounds. The winner (or set of winners) is determined by a function of the outcomes of all the rounds. Note that the outcomes of the individual matches are determined probabilistically, whereas once these outcomes have been determined, the winners of the tournament are found deterministically.

Bounded tournament: A tournament where the number of rounds is bounded from above by a constant.

Bounded history tournament: A tournament where the set of matches played in the current round depends only on the outcomes of the previous b rounds instead of all previous rounds, for some constant b.

Cup tournament: A single-elimination competition (or knockout tournament [51]) over a complete binary tree where each entrant[3] plays a sequence of matches head-to-head; the winner is the entrant that is left undefeated. The United States' men's and women's NCAA (college) Basketball Tournaments and most tennis majors fall into this category.

[3] We use the term entrant in this paper because we can imagine a tournament made up of individuals or teams.