Journal of Applied Logic 13 (2015) 605–627

Contents lists available at ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Using inductive reasoning for completing OCF-networks

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A R T I C L E   I N F O

Article history:
Available online 18 March 2015

Keywords:
Ranking functions
Networks
Conditionals
Inductive reasoning
System Z^+
c-Representations

A B S T R A C T

OCF-networks provide the possibility to combine qualitative information expressed by rankings of (conditional) formulas with the strong structural information of a network, in this respect being a qualitative variant of the better known Bayesian networks. Like for Bayesian networks, a global ranking function can be calculated quickly and efficiently from the locally distributed information, whereas the latter significantly reduces the exponentially high complexity of the semantical ranking approach. This qualifies OCF-networks for applications. However, in practical applications the provided ranking information may not be in the format needed to be represented by an OCF-network, or some values may be simply missing. In this paper, we present techniques for filling in the missing values using methods of inductive reasoning and we elaborate on formal properties of OCF-networks.

1. Introduction

Successful reasoning in an uncertain environment usually depends crucially on a suitable and appropriate semantical framework that allows for rich and meaningful representation of the problem domain, on the one hand, and leaving enough semantical room for handling exceptions and nonmonotonic phenomena, on the other hand. A most common, popular and widely-used framework which provides such possibilities is probability theory, another is provided by the theory of *ordinal conditional function (OCF)* [17], also known under the name of *ranking functions* which bestow an ordering of implausibility upon the set of possible worlds. Being essentially qualitative, they nevertheless share some nice features with probabilities. A major feature of ranking functions is that they provide proper interpretations for defeasible rules *If* A *then plausibly/usually* B interpreted as non-material, meaningful conditionals $(B|A)$ by encoding plausible relationships between the respective antecedents (premises) A and consequents (conclusions) B.

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http://dx.doi.org/10.1016/j.jal.2015.03.006

Unfortunately, typical semantical approaches have to take every single element of an exponentially large set of elementary events or possible worlds into account, which also yields an exponentially high complexity. This disadvantage leads to the idea of partitioning these sets such that only small sets have to be handled. Here graphical structures like the well-established Bayesian networks [14] have been found to be extremely useful. This approach is closely connected with causal interpretations along the edges, considering the parents of a vertex, respectively variable, as its (common) causes. Similar approaches have been proposed for ranking functions [1,6] which allow for local storing of conditional ranking values at the network's vertices and efficient combination thereof to provide a global view. As with Bayesian networks, this local information encodes the plausibility of a single vertex variable given all its parents. This information, however, may not be available in application scenarios, where, for example, an expert or the user may be able to give the plausibility of a vertex given each of the parents but not of the whole set and therefore the local information may be missing.

In this paper, we apply methods of inductive reasoning like System Z^+ [5] and c-representations [8] to such scenarios, taking into account the local information present and calculate (complete) rankings for the respective subgraph, that is, a vertex and its parents. From this semantical information the missing values from the child node will then be extracted. Again, similar approaches have been presented for Bayesian networks by making use of the maximum entropy principle [12,13,16]. Indeed, the maximum entropy distribution is a probabilistic c-representation for the given knowledge base [8], and for the OCF framework, inferences based on c-representations have also proved to satisfy all major postulates of nonmonotonic reasoning [7]. Therefore we make use of high quality semantical methods to exploit the given partial information in an optimal way. We then use this method of local computation for knowledge bases of single-elementary conditionals, illustrating how the methods of inductive reasoning can be used to construct local tables for a network set up from the knowledge base using an algorithm from [6].

This paper is an extended version of the paper [9]; in particular, we elaborate on formal properties of OCFs and OCF-networks in much more detail, making conditional independence and a formula for total (conditional) ranks for OCFs explicit. We show, as presupposed in [1,6], that the local directed Markov property holds and that the global rankings of the stratified OCF coincide with the local rankings in the tables (Theorems 1, 2). The application of our methods to knowledge bases also broadens the previous papers' perspectives to a more generic view.

This paper is organised as follows: We introduce the formal preliminaries in Section 2. In Section 3 we recall ranking functions, elaborating on a notion of conditional independence and the ranking analogy of total (conditional) probability. The method presented in this article relies on inductive reasoning, the used approaches, namely System Z^+ and c-representations, are introduced in Section 4. In Section 5 we recall the concept of OCF-networks and discuss formal properties of this concept. We then discuss why local information may not be available or not in the needed format for all vertices of the network and show how to solve this problem with the presented approaches of inductive reasoning in Section 6. This approach is then generalised in Section 7 to the case where no network but a knowledge base is present in order to efficiently use local information for global reasoning. For this we recall a graph generation algorithm from [6], and we discuss whether inconsistency of the knowledge base is related to acyclicity of the graph or not. We end this section by considering admissibility regarding the generating local knowledge base. Section 8 then transfers the results of the former sections from the quantitative to the purely qualitative case. We discuss the results in Section 9 and finally conclude in Section 10.

2. Preliminaries

Let $\Sigma = \{V_1, \ldots, V_p\}$ be a set of propositional atoms and a *literal* a positive or negative atom representing variables in their positive resp. negated form; for a specific, nevertheless undetermined, outcome of V_i, we write $\dot{v}_i \in \{v_i, \bar{v}_i\}$.