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Measuring inaccuracy of uncertain doxastic states in many-valued logical systems

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ABSTRACT

I will propose an alternative philosophical approach to the representation of uncertain doxastic states. I will argue that the current account of measuring inaccuracy of uncertain doxastic states is inadequate for Belnap's four-valued logic. Specifically, a situation can be found in which either an inaccuracy measure returns a completely wrong result or an agent's inaccuracy score is inadequate relative to the mistake in her doxastic attitude. This will motivate an alternative representation of uncertain doxastic states based on ordered pairs. I will describe a possible inaccuracy measure that is suitable for ordered pairs, and I will show that it has all the qualities that are required for an inaccuracy measure to be legitimate. Finally, I will introduce conditions of rationality for uncertain doxastic states represented by ordered pairs.

1. Introduction

Imagine an agent who uses single real-valued numbers from the unit interval to represent her credences. For example, she represents her credence that a fair coin lands heads by 0.5. Imagine that she is also concerned about the inaccuracy of her credences. I claim that for such an agent the current approach to measuring the inaccuracy of uncertain doxastic states is inappropriate when Belnap's four-valued logic (Bel_4) is considered.

The inaccuracy of an agent's credence in a proposition X at a possible world w is expressed as a mathematical function, for example a squared difference, of an agent's credence in X and the ideal credence in X at that w. Throughout the paper, I will use adjectives ideal, omniscient, and vindicated for credence functions, epistemic states, etc. interchangeably. I will postulate some properties that, I believe, ideal credences should have. I will then argue that representing ideal credences in Bel_4 by single values from the unit interval cannot satisfy these conditions simultaneously. As a consequence, there will be a situation in which either an inaccuracy measure will return a wrong result or an agent's credence will be assigned an inadequate inaccuracy score.

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This issue will motivate the introduction of an alternative philosophical approach to the representation of uncertain doxastic states. I will use an ordered pair of numbers instead of a single number to numerically represent an agent's uncertain doxastic state with respect to a proposition.[1] I will then modify the Brier score, which is the most popular of the inaccuracy measures when uncertain doxastic states are represented by single numerical values, to develop a measure $\mathscr{B}^*$ that can be used to measure the inaccuracy of uncertain doxastic states represented by ordered pairs. I will show that $\mathscr{B}^*$ is a legitimate inaccuracy measure. It means that $\mathscr{B}^*$ has all the properties that legitimate inaccuracy measures are required to have. $\mathscr{B}^*$ will be enough to show that the system of ordered pairs meets all the requirements for ideal credences and that this system is appropriate for measuring the inaccuracy of uncertain doxastic states in Bel_4. Although I will be using the Brier score and the modified Brier score $\mathscr{B}^*$ throughout the paper, the conclusion and arguments are not restricted to these measures, and can also be made with other legitimate inaccuracy measures and their modifications for ordered pairs.

To develop the theory further, I will introduce and argue for conditions of rationality for uncertain doxastic states represented by ordered pairs. I will use the structure of the argument that epistemic utility theory (EUT) uses to justify Probabilism to build up a valid argument for conditions of rationality for ordered pairs. I will concentrate on Bel_4 in my argument, but it should be understood as a template how to interpret general mathematical results epistemically when ordered pairs are used, and how to construct similar arguments using ordered pairs for different many-valued systems like Priest's Logic of Paradox or Kleene's three-valued logic.

The structure of the paper is as follows. Section 2 discusses issues of measuring the inaccuracy of uncertain doxastic states in Bel_4. Section 3 introduces the representation of uncertain doxastic states by ordered pairs and gives an account of how to measure their inaccuracy with $\mathscr{B}^*$. It is argued that the system of ordered pairs together with $\mathscr{B}^*$ is appropriate to measure the inaccuracy of uncertain doxastic states in Bel_4. Section 4 introduces conditions of rationality for uncertain doxastic states represented by ordered pairs in Bel_4. In Appendix A, I present the relevant mathematical results.

2. Measuring inaccuracy in Belnap's four-valued logic

Epistemic utility theory (EUT) is a theory that provides a non-pragmatic, that is, purely epistemic justification of epistemic norms. It is an alternative, but not necessarily a rival, to Dutch Book arguments. EUT claims that the fundamental epistemic value is how well an agent's doxastic states represent the world, which is how accurate they are. It then characterizes the legitimate measures of accuracy, and given such measures arguments are made for epistemic norms [9]. EUT was originally developed to justify Probabilism [5] and later on used, for example, to justify norms for qualitative belief states [2], the Principal Principle [8], or the Principle of Indifference [10]. As I said, EUT considers accuracy to be the fundamental source of epistemic value for credences, but in what follows, it will be easier to talk of inaccuracy of credences and measures of that quantity rather than accuracy and measures of that. It does not introduce any big change. Accuracy will simply be inaccuracy with the sign reversed. Thus, if I is a measure of inaccuracy, $-I$ is a measure of accuracy. For example, the Brier score $\mathscr{B}$ and the new measure $\mathscr{B}^*$ are inaccuracy measures. Accuracy measures are then $-\mathscr{B}$ and $-\mathscr{B}^*$.

EUT represents an agent's uncertain doxastic state by a credence function $c : \mathcal{F} \to [0, 1]$, where $\mathcal{F}$ is a finite set, usually an algebra, of propositions to which an agent assigns credences. $\mathcal{F}$ is called an opinion set. EUT then measures how inaccurate an agent's doxastic state c is. To do that, EUT takes inaccuracy of an agent's individual credences in elements of $\mathcal{F}$ at a possible world $w \in W_{\mathcal{F}}$. Where $W_{\mathcal{F}}$ is a set of possible

[1] A similar formal approach was taken by some computer scientist in early 1990's when they tried to develop a representation of doxastic attitudes for many valued logical systems [3].