



Borderlines and probabilities of borderlines: On the interconnection between vagueness and uncertainty



Jonathan Lawry

Department of Engineering Mathematics, University of Bristol, Bristol BS8 1UB, UK

ARTICLE INFO

Article history:

Received 14 January 2015

Received in revised form 24 July 2015

Accepted 11 August 2015

Available online 28 September 2015

Keywords:

Vagueness

Uncertainty

Borderline cases

Three-valued valuations

ABSTRACT

We describe an integrated approach to vagueness and uncertainty within a propositional logic setting and based on a combination of three valued logic and probability. Three valued valuations are employed in order to model explicitly borderline cases and in this context we give an axiomatic characterisation of two well known three valued models; supervaluations and Kleene valuations. We then demonstrate the close relationship between Kleene valuations and a sub-class of supervaluations. Belief pairs are lower and upper measures on the sentences of the language generated from a probability distribution defined over a finite set of three valued valuations. We describe links between these measures and other uncertainty theories and we show the close relationship between Kleene belief pairs and a sub-class of supervaluation belief pairs. Finally, a probabilistic approach to conditioning is explored within this framework.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

There is a highly interconnected relationship between vagueness and uncertainty. It is not just that vagueness occurs in conjunction with epistemic uncertainty but also that linguistic uncertainty is integral to vague propositions themselves. The latter refers to uncertainty about the definition or interpretation of concepts in natural language and is a natural result of the empirical manner in which language is learnt. Lawry [13] and Lassiter [12] argue that this form of uncertainty is epistemic in nature and can be modelled probabilistically. In this case, the blurred boundary of a vague category can be modelled by probability defined over possible precise boundaries. There is nonetheless an important distinction between blurred boundaries and the explicit identification of borderline cases. Indeed the latter does not refer to epistemic uncertainty at all but instead results from a non-Boolean truth model. For example, given an exact value for Ethel's height it might be certain that she is borderline short. Vagueness is not only the result of linguistic uncertainty or of borderline cases but comprises of at least both of these features. Furthermore,

E-mail address: j.lawry@bris.ac.uk.

vague predicates are everywhere embedded in our statements and beliefs about the world. Consequently, to assess such beliefs we must consider vagueness in conjunction with epistemic uncertainty about the state of the world. This requires an integrated approach capturing both uncertainty about the world and linguistic uncertainty about the conventions of language, together with non-Boolean truth models resulting from more flexible category representation.

In this paper we investigate these ideas in a propositional logic setting by combining probability and three valued valuations i.e. taking truth values true, borderline or false. Initially, we adopt an axiomatic approach and consider what properties should be satisfied by three valued valuations if they are to appropriately represent explicitly borderline cases, and following on from this we then investigate the relationship between two different types of valuations. More specifically, we show that there is a strong relationship between Kleene valuations and a sub-type of supervaluations over a restricted set of formulae of the language. As a means of combining epistemic uncertainty and explicitly borderline cases we will introduce belief pairs in the form of lower and upper measures on the sentences of the language. These are generated from probability distributions defined over three valued valuations. More formally, the lower measure of a sentence will be taken as corresponding to the probability that it is true, and the upper measure as corresponding to the probability that it is not false. We introduce different types of belief pairs based on different underlying three valued truth models, and we investigate some of the relationships between them. We then extend these ideas so as to consider conditional beliefs based on probabilistic conditioning over three valued truth models.

An overview of the paper is as follows: In Section 2 we introduce a generic definition of three valued valuation in a propositional logic setting and give Kleene valuations and supervaluations as distinct examples. Section 3 proposes a number of axiomatic properties which we might require a suitable three valued valuation model to satisfy. We discuss the reasonableness of these properties and provide characterisations of both Kleene valuations and supervaluations. The notion of a vagueness ordering of valuations is discussed in Section 4 and a candidate partial ordering is proposed. These ideas are then used as the basis of an argument against Łukasiewicz valuations as a model of borderline cases. Belief pairs are introduced in Section 6 where we exploit the results in Section 5 in order to demonstrate the relationship between Kleene and supervaluation belief pairs. Furthermore, we consider the special case in which uncertainty only concerns the level of vagueness of the language. In Section 7 we then outline a model of conditional belief within our proposed framework. Finally, in Section 8 we give some discussion and conclusions.

The main contributions of this paper are as follows: Firstly we give axiomatic characterisations of both supervaluations and Kleene valuations as special cases of a very general class of three valued truth functions. This helps to make explicit the assumptions about the behaviour of borderline cases which is implicit in each case. Secondly, we clarify the relationship between Kleene valuations and a sub-class of supervaluations called complete bounded supervaluations. It is shown that these two types of valuations are equal on the subset of sentences in negated normal form which do not involve both a propositional variable and its negation. Consequently, we have identified a functional class of supervaluations,¹ which are similar to Kleene valuations but which preserve classical logic equivalences and tautologies. Thirdly, we extend these results to belief pairs consisting of lower and upper belief measures generated from a probability distribution defined over a finite set of three valued valuations. More specifically, we show that complete bounded supervaluation belief pairs coincide with Kleene belief pairs for the same class of sentences described above. Finally, we investigate conditional belief pairs as generated by conditional probabilities defined over a finite set of three valued valuations. This is rather a novel approach to conditioning for non-classical logic, and is quite distinct from the more usual implication operators defined for many valued logics. We prove a number of results for conditional supervaluation and Kleene belief pairs under different assumptions. In some cases the work

¹ Although supervaluations are never truth-function they can be functional in a weaker sense. More details are given later in the paper.

Download English Version:

<https://daneshyari.com/en/article/6424883>

Download Persian Version:

<https://daneshyari.com/article/6424883>

[Daneshyari.com](https://daneshyari.com)