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The Witten–Reshetikhin–Turaev invariant for links in finite order mapping tori I

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ABSTRACT

We state Asymptotic Expansion and Growth Rate conjectures for the Witten–Reshetikhin–Turaev invariants of arbitrary framed links in 3-manifolds, and we prove these conjectures for the natural links in mapping tori of finite-order automorphisms of marked surfaces. Our approach is based upon geometric quantisation of the moduli space of parabolic bundles on the surface, which we show coincides with the construction of the Witten–Reshetikhin–Turaev invariants

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Gauge theory
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Growth rate conjecture

using conformal field theory, as was recently completed by Andersen and Ueno.

1. Introduction

In this paper we study the asymptotic expansion of the Witten–Reshetikhin–Turaev (WRT) invariants of certain 3-manifolds with links, building on the work [4,5], which also used the geometric construction of the WRT-TQFT via the geometric quantisation of moduli spaces of flat connections on surfaces as first considered by Axelrod–Della Pietra–Witten [14], Hitchin [44] and further explored by the first named author to prove asymptotic faithfulness [3]. For references concerning the study of the large level asymptotics of the WRT quantum invariants of closed 3-manifolds see the references in [4]. Let us here first present a generalisation of the Asymptotic Expansion Conjecture to pairs consisting of a general closed oriented 3-manifold together with an embedded oriented framed link, labelled by level dependent labels.

1.1. The asymptotic expansion and growth rate conjectures

The quantum invariants and their associated Topological Quantum Field Theories were proposed in Witten's seminal paper [79] on quantum Chern–Simons theory with a general compact simple simply-connected Lie group K, and subsequently constructed by Reshetikhin and Turaev [64,65,75] for $K = \mathrm{SU}(2)$ and then for $K = \mathrm{SU}(N)$ by Wenzl and Turaev in [76,77]. These TQFTs were also constructed from skein theory by Blanchet, Habegger, Masbaum and Vogel in [25,26] for $K = \mathrm{SU}(2)$ and for $K = \mathrm{SU}(N)$ in [24]. We will denote these TQFTs for $K = \mathrm{SU}(N)$ by $Z_N^{(k)}$. The WRT-TQFT associated to a general simple simply-connected Lie group K will be denoted by $Z_K^{(k)}$, e.g. $Z_N^{(k)} = Z_{\mathrm{SU}(N)}^{(k)}$.

The label set of the WRT-TQFT $Z_K^{(k)}$ theory is given as

$$\Lambda_K^{(k)} = \{\, \lambda \in P_+ \,|\, 0 \le \langle \theta, \lambda \rangle \le k \,\}, \tag{1.1}$$

where P_+ is the set of dominant integral weights of $\mathfrak{k}$, the Lie algebra of K. Here $\langle\ ,\ \rangle$ is the normalized Cartan–Killing form defined to be a constant multiple of the Cartan–Killing form such that $\langle \theta, \theta \rangle = 2$, for the longest root θ of $\mathfrak{k}$. We will use $\langle\ ,\ \rangle$ at various places throughout the text to identify weights and coweights.

Let X be an oriented closed 3-manifold and let L be a framed link contained in X. For notational purposes pick an ordering of the components of $L = L_1 \cup \ldots \cup L_n$. Let $\overline{\lambda}^{(k)} = (\lambda_1^{(k)}, \ldots, \lambda_n^{(k)})$, be a labelling of the components of L which is k-dependent (possibly only for k forming a sub-sequence of $\mathbb{N}$). In fact, throughout this paper we will restrict to the simple example $\lambda_i^{(k)} = \lambda_i s$ for k-independent $\lambda_i \in \Lambda_N^{(k_0)}$, with $k = sk_0$