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Symmetric Lie superalgebras and deformed quantum Calogero–Moser problems

A.N. Sergeev [a,b], A.P. Veselov [c,d,*]

[a] Department of Mathematics, Saratov State University, Astrakhanskaya 83, Saratov 410012, Russia
[b] National Research University Higher School of Economics, 20 Myasnitskaya Ulitsa, Moscow 101000, Russia
[c] Department of Mathematical Sciences, Loughborough University, Loughborough LE11 3TU, UK
[d] Moscow State University, Moscow 119899, Russia

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ABSTRACT

The representation theory of symmetric Lie superalgebras and corresponding spherical functions are studied in relation with the theory of the deformed quantum Calogero–Moser systems. In the special case of symmetric pair $\mathfrak{g} = \mathfrak{gl}(n, 2m), \mathfrak{k} = \mathfrak{osp}(n, 2m)$ we establish a natural bijection between projective covers of spherically typical irreducible $\mathfrak{g}$-modules and the finite dimensional generalised eigenspaces of the algebra of Calogero–Moser integrals $\mathfrak{D}_{n,m}$ acting on the corresponding Laurent quasi-invariants $\mathfrak{A}_{n,m}$.

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* Corresponding author.
E-mail addresses: SergeevAN@info.sgu.ru (A.N. Sergeev), A.P.Veselov@lboro.ac.uk (A.P. Veselov).

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1. Introduction

In 1964 Berezin et al. [3] made an important remark that the radial part of the Laplace–Beltrami operator on the symmetric space $X = SL(n)/SO(n)$

$$L = \Delta + \sum_{i<j}^{n} \coth(x_i - x_j)(\partial_i - \partial_j)$$

is conjugated to the quantum Hamiltonian

$$H = \Delta + \sum_{i<j}^{n} \frac{1}{2\sinh^2(x_i - x_j)}$$

describing the pairwise interacting particle on the line.

This was probably the first recorded observation of the connection between the theory of symmetric spaces and the theory of what later became known as Calogero–Moser, or Calogero–Moser–Sutherland (CMS), integrable models [34]. Olshanetsky and Perelomov suggested a class of generalisations of CMS systems related to any root system and showed that the radial parts of all irreducible symmetric spaces are conjugated to some particular operators from this class [20]. The joint eigenfunctions of the corresponding commutative algebras of quantum integrals are zonal spherical functions. In the A_n case this leads to an important notion of the Jack polynomials introduced by H. Jack independently around the same time [14].

The discovery of the Dunkl operator technique led to an important link of the CMS systems with the representation theory of Cherednik algebras, see Etingof's lectures [10].

It turned out that there are other integrable generalisations, which have only partial symmetry and called deformed CMS systems [8]. Their relation with symmetric superspaces was first discovered by one of the authors in [26] and led to a class of such operators related to the basic classical Lie superalgebras, which was introduced in [27].

In this paper we develop this link further to study the representation theory of symmetric Lie superalgebras and the related spherical functions. Such Lie superalgebra is a pair $(\mathfrak{g}, \theta)$, where $\mathfrak{g}$ is a Lie superalgebra and θ is an involutive automorphism of $\mathfrak{g}$. It corresponds to the symmetric pair $X = (\mathfrak{g}, \mathfrak{k})$, where $\mathfrak{k}$ is θ-invariant part of $\mathfrak{g}$ and can be considered as an algebraic version of the symmetric superspace G/K.