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Algebraic K-theory of group rings and the cyclotomic trace map

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ABSTRACT

We prove that the Farrell–Jones assembly map for connective algebraic K-theory is rationally injective, under mild homological finiteness conditions on the group and assuming that a weak version of the Leopoldt–Schneider conjecture holds for cyclotomic fields. This generalizes a result of Bökstedt, Hsiang, and Madsen, and leads to a concrete description of a large direct summand of $K_n(\mathbb{Z}[G]) \otimes_{\mathbb{Z}} \mathbb{Q}$ in terms of group homology. In many cases the number theoretic conjectures are true, so we obtain rational injectivity results about assembly maps, in particular for Whitehead groups, under homological finiteness assumptions on the group only. The proof uses the cyclotomic trace map to topological cyclic homology, Bökstedt–Hsiang–Madsen's functor C, and new general isomorphism and injectivity results about the assembly maps for topological Hochschild homology and C.

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Contents

1. Introduction

The algebraic K-theory groups $K_n(\mathbb{Z}[G])$ of the integral group ring of a discrete group G have far-reaching geometric applications to the study of manifolds with fundamental group G and of sufficiently high dimension. One of the most famous and important manifestations of this phenomenon is given by the Whitehead group $Wh(G)$, which is the quotient of $K_1(\mathbb{Z}[G]) = GL(\mathbb{Z}[G])_{\mathrm{ab}}$ by the subgroup generated by the units in $\mathbb{Z}[G]$ of the form $\pm g$ with $g \in G$. By the celebrated s-Cobordism Theorem, $Wh(G)$ completely classifies the isomorphism classes of h-cobordisms over any closed, connected manifold M with $\dim(M) \geq 5$ and $\pi_1(M) \cong G$.

A well-known and still open conjecture, which we review below, predicts that the Whitehead group of any torsion-free group vanishes. However, if a group G has torsion, then in general $Wh(G)$ is not trivial. For example, the structure of the Whitehead groups of finite groups G is well understood (see [87]), and if C is any finite cyclic group of order $c \notin \{1, 2, 3, 4, 6\}$, then $Wh(C)$ is not zero, not even after tensoring with the rational numbers $\mathbb{Q}$.

One of the main consequences of this work is the following theorem about Whitehead groups of infinite groups with torsion. Its conclusion says that rationally the Whitehead groups $Wh(H)$ of all finite subgroups H of G contribute to $Wh(G)$, and only the obvious relations between these contributions hold. Its assumption is a very weak and natural homological finiteness condition only up to dimension two. As we explain in Section 2, it is satisfied by many geometrically interesting groups, such as outer automorphism groups of free groups and Thompson's group T, thus yielding the first known results about the