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Derived equivalences for hereditary Artin algebras

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We study the role of the Serre functor in the theory of derived equivalences. Let $\mathcal{A}$ be an abelian category and let $(\mathcal{U}, \mathcal{V})$ be a t-structure on the bounded derived category $D^b\mathcal{A}$ with heart $\mathcal{H}$. We investigate when the natural embedding $\mathcal{H} \to D^b\mathcal{A}$ can be extended to a triangle equivalence $D^b\mathcal{H} \to D^b\mathcal{A}$. Our focus of study is the case where $\mathcal{A}$ is the category of finite-dimensional modules over a finite-dimensional hereditary algebra. In this case, we prove that such an extension exists if and only if the t-structure is bounded and the aisle $\mathcal{U}$ of the t-structure is closed under the Serre functor.

Contents

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1. Introduction

Let $\mathcal{A}$ be an abelian category. In the bounded derived category $D^b\mathcal{A}$, we consider the full subcategory $D^{\leq 0}_{\mathcal{A}}$ of all objects X such that $H^nX = 0$ for all $n > 0$, and the full subcategory $D^{\geq 0}_{\mathcal{A}}$ of all objects X such that $H^nX = 0$ for all $n < 0$. We can recover $\mathcal{A}$ (up to equivalence) as $D^{\leq 0}_{\mathcal{A}} \cap D^{\geq 0}_{\mathcal{A}}$.

A pair $(\mathcal{U}, \mathcal{V})$ of full subcategories of $D^b\mathcal{A}$ with properties similar to the pair $(D^{\leq 0}_{\mathcal{A}}, D^{\geq 0}_{\mathcal{A}})$ given above is called a t-structure (see [8] or §3.3). The definitions are chosen so that $\mathcal{U} \cap \mathcal{V}$ is an abelian category, called the *heart* of $(\mathcal{U}, \mathcal{V})$.

We will say that the abelian categories $\mathcal{A}$ and $\mathcal{B}$ are *derived equivalent* if there is a triangle equivalence $F : D^b\mathcal{A} \to D^b\mathcal{B}$. Using F, one can transfer the standard t-structure $(D^{\leq 0}_{\mathcal{B}}, D^{\geq 0}_{\mathcal{B}})$ on $D^b\mathcal{B}$ across to a t-structure $(\mathcal{U}, \mathcal{V})$ on $D^b\mathcal{A}$, whose heart $\mathcal{U} \cap \mathcal{V}$ is equivalent to $\mathcal{B}$.

However, this situation is not representative for the general situation. Indeed, even though it is possible, for any t-structure $(\mathcal{U}, \mathcal{V})$ on $D^b\mathcal{A}$, to extend the natural embedding $\mathcal{H} \to D^b\mathcal{A}$ of the heart to a triangle functor $F : D^b\mathcal{H} \to D^b\mathcal{A}$, there might be no choice of F which is an equivalence. If there is such a choice $F : D^b\mathcal{H} \to D^b\mathcal{A}$, we will say that the t-structure $(\mathcal{U}, \mathcal{V})$ *induces a derived equivalence* (see Definition 4.1). We will discuss this in §4.1 (based on necessary and sufficient conditions given in [8]).

One necessary condition for a t-structure $(\mathcal{U}, \mathcal{V})$ to induce a derived equivalence is that the t-structure must be *bounded*, meaning that $\cup_{n \in \mathbb{Z}} \mathcal{U}[n] = D^b\mathcal{A} = \cup_{n \in \mathbb{Z}} \mathcal{V}[n]$. Here, we write $X[n]$ for the n-fold suspension of X.