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Normal forms and embeddings for power-log transseries ☆

P. Mardešić [a], M. Resman [b], J.-P. Rolin [a,*], V. Županović [b]

[a] Université de Bourgogne, Départment de Mathématiques, Institut de Mathématiques de Bourgogne, B.P. 47 870, 21078-Dijon Cedex, France
[b] University of Zagreb, Department of Applied Mathematics, Faculty of Electrical Engineering and Computing, Unska 3, 10000 Zagreb, Croatia

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ABSTRACT

Dulac series are asymptotic expansions of first return maps in a neighborhood of a hyperbolic polycycle. In this article, we consider two algebras of power-log transseries (generalized series) which extend the algebra of Dulac series. We give a formal normal form and prove a formal embedding theorem for transseries in these algebras.

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* Corresponding author.
E-mail address: rolin@u-bourgogne.fr (J.-P. Rolin).

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1. Introduction and main results

1.1. Description of the problem and motivation

In the study of discrete dynamical systems, two problems are particularly important: the search of a *normal form* and the *embedding* problem. The search of a normal form means a definite process of choosing a representative of the class of conjugacy of the original system under topological, smooth or holomorphic conjugacies. This representative should be simpler than the original one. The embedding problem consists in finding a vector field such that the original system is the time-one map of its flow. These two problems are obviously connected: it should be easier to embed a normal form in a flow than the original system itself.

The study of holomorphic or real analytic systems at the origin of the ambient space naturally leads to the problems of *formal normal form* and *formal embedding*. These questions are discussed in detail, e.g., in [8, Chapter I, Sections 3 and 4]. The case of dimension 1 is well understood. Consider, for example, a *parabolic* formal series

$$f(z) = z + a_1 z^{p+1} + z^{p+1}\varepsilon(z),$$

where $a_1 \in \mathbb{C}^*$, $\varepsilon(z) \in \mathbb{C}[[z]]$ and $\varepsilon(0) = 0$. It is well known that the formal conjugacy class of f has a polynomial representative $f_0(z) = z + z^{p+1} + az^{2p+1}$, where the *residual*