

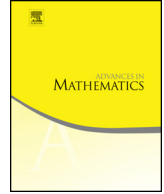


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



The six operations in equivariant motivic homotopy theory



Marc Hoyois¹

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, USA

ARTICLE INFO

Article history:

Received 20 November 2015
Received in revised form 3 September 2016
Accepted 25 September 2016
Available online 30 September 2016
Communicated by A. Blumberg

MSC:
14F42
55P91

Keywords:

Motivic homotopy theory
Equivariant homotopy theory
Algebraic stacks

ABSTRACT

We introduce and study the homotopy theory of motivic spaces and spectra parametrized by quotient stacks $[X/G]$, where G is a linearly reductive linear algebraic group. We extend to this equivariant setting the main foundational results of motivic homotopy theory: the (unstable) purity and gluing theorems of Morel–Voevodsky and the (stable) ambidexterity theorem of Ayoub. Our proof of the latter is different than Ayoub’s and is of interest even when G is trivial. Using these results, we construct a formalism of six operations for equivariant motivic spectra, and we deduce that any cohomology theory for G -schemes that is represented by an absolute motivic spectrum satisfies descent for the cdh topology.

© 2016 Elsevier Inc. All rights reserved.

Contents

1.	Introduction	198
1.1.	Equivariant homotopy theory	199
1.2.	The formalism of six operations	201
1.3.	Summary of the construction	204

E-mail address: hoyois@mit.edu.

URL: <http://math.mit.edu/~hoyois/>.

¹ This work was partially supported by the National Science Foundation under grant DMS-1508096.

1.4.	Some applications	207
1.4.1.	Motivic Wirthmüller and Adams isomorphisms	207
1.4.2.	An equivariant Lefschetz trace formula	208
1.4.3.	Cdh descent for the homotopy K -theory of tame stacks	208
1.5.	Acknowledgments	208
1.6.	Notation and terminology	209
2.	Some equivariant geometry	209
2.1.	Invariant subschemes	209
2.2.	Quasi-affine and quasi-projective morphisms	212
2.3.	Linearly reductive groups	214
2.4.	Affine resolutions	215
2.5.	Lifting of smooth quasi-sections	217
2.6.	Linearizations	219
2.7.	Tame group schemes	220
3.	Unstable equivariant motivic homotopy theory	221
3.1.	Preliminaries	221
3.2.	Homotopy invariance	222
3.3.	Nisnevich excision	224
3.4.	Equivariant motivic spaces	226
3.5.	Smooth closed pairs	227
4.	Functoriality	233
4.1.	Easy results	233
4.2.	Exactness properties of pushforwards	237
4.3.	Gluing	242
5.	Ambidexterity for smooth projective morphisms	245
5.1.	Pointed equivariant motivic spaces	245
5.2.	Spheres, Thom spaces, and purity	247
5.3.	The Pontryagin–Thom collapse	250
6.	Stable equivariant motivic homotopy theory	262
6.1.	Equivariant motivic spectra	263
6.2.	The exceptional functors	269
6.3.	Descent properties	275
	References	278

1. Introduction

The goal of this paper is to develop the formalism of six operations in stable equivariant motivic homotopy theory. An equivariant version of motivic homotopy theory was first considered by Voevodsky in [9] and played a small part in his proof of the Bloch–Kato conjecture [49], more precisely in the construction of symmetric power functors on the \mathbb{A}^1 -homotopy category [48, §2.1]. A variety of definitions of equivariant motivic homotopy theory were later proposed by several authors: by Hu, Kriz, and Ormsby [27], by Herrmann [22,23], by Heller, Krishna, and Østvær [20], and by Carlsson and Joshua [5]. In these approaches, equivariant motivic homotopy theory is a setting in which to study invariants of smooth G -schemes with some specific properties, for G an algebraic group. We will discuss in §1.3 below how these approaches relate to the one developed in this paper. In any case, our starting point is somewhat different: we view equivariant homotopy theory as a natural extension of parametrized homotopy theory, and the formalism of six operations serves as a guiding principle in our definitions.

Download English Version:

<https://daneshyari.com/en/article/6425048>

Download Persian Version:

<https://daneshyari.com/article/6425048>

[Daneshyari.com](https://daneshyari.com)