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Intermediate extension of Chow motives of Abelian type

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ABSTRACT

In this article, we give an unconditional construction of a motivic analogue of the intermediate extension in the context of Chow motives of Abelian type. Our main application concerns intermediate extensions of Chow motives associated to Kuga families to the Baily–Borel compactification of a Shimura variety.

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Contents

0. Introduction 516
1. Gluing of semi-primary categories 523

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2. The abstract theory of intermediate extensions 531
3. The motivic picture 545
4. Gluing of motives 548
5. Construction of semi-primary categories of Chow motives 557
6. Compatibility with certain direct and inverse images 562
7. Compatibility with realizations 565
8. Applications 581
References 598

0. Introduction

A profound conjecture concerning the category of motives over some base X predicts the existence of a t-structure, all of whose realizations are compatible with the so-called *perverse* t-structure. This structure would in particular allow for the construction of the *intermediate extension* to X of any Chow motive over an open sub-scheme U of X, canonically up to (unipotent) automorphisms restricting to the identity on U.

Unfortunately, the assumption concerning the t-structure is extremely hypothetical: for X equal to the spectrum of a field of characteristic zero, it would imply both Grothendieck's standard conjectures and Murre's filtration conjecture [9].

The aim of this paper is to establish the theory of intermediate extensions. Its ingredients are radically different from t-structures. It is based on two key notions: *weight structures* à la Bondarko, and *semi-primary categories* à la André–Kahn. This approach is *formula free*. Above all, it is *unconditional*, once certain geometric conditions are satisfied.

In order to illustrate our results, let us discuss their implications in a context which lends itself to important arithmetic applications. Let M^L be a *pure Shimura variety*, and assume that the group L to which it is associated, is *neat*, which implies that M^L is smooth over the *reflex field* E. The variety M^L is the target of proper, smooth morphisms

$$\pi : M^K \longrightarrow M^L ,$$

induced by a morphism of *Shimura data* $(P, \mathfrak{X}) \to (G, \mathfrak{H})$, which identifies G with the maximal reductive quotient of P. We make a mild technical assumption on the Shimura data $(G, \mathfrak{H})$, namely that they satisfy [47, Condition (3.1.5)] (this condition will be recalled later). The source of π is a *Kuga variety*, *i.e.*, a *mixed Shimura variety* admitting the structure of a torsor under an Abelian scheme over M^L. (Note that we admit the case $\pi = \mathrm{id}_{M^L}$, *i.e.*, the Abelian scheme may be of relative dimension zero.) Fix one such π. The scheme M^L being regular, and π proper and smooth, the motive

$$\pi_* \mathbb{1}_{M^K}$$

belongs to the category $CHM^s(M^L)_{\mathbb{Q}}$ of *smooth Chow motives* over M^L [39]. Fix an extension F of $\mathbb{Q}$, and a direct factor N of $\pi_* \mathbb{1}_{M^K}$, viewed as an object of the category

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