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Multiplier ideal sheaves associated with weights of log canonical threshold one

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A B S T R A C T

In this article, we will characterize the multiplier ideal sheaves associated with weights of log canonical threshold one by restricting the weights to complex regular surface.

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1. Introduction

1.1. Background

Let $\Omega \subset \mathbb{C}^n$ be a domain and $o \in \Omega$ the origin. Let u be a plurisubharmonic function on Ω. The multiplier ideal sheaf $\mathscr{I}(u)$ is defined to be the sheaf of germs of holomorphic functions f such that $|f|^2e^{-2u}$ is locally integrable (see [1]). Here, u is regarded as the weight of $\mathscr{I}(u)$. The *Lelong number* of u at o is defined to be

$$\nu(u, o) := \sup\{\gamma \geq 0 | u(z) \leq \gamma \log |z| + O(1) \text{ near } o\}.$$

When the Lelong number satisfies $\nu(u, o) < 1$, Skoda showed that $\mathscr{I}(u)_o = \mathcal{O}_n$ (see [1]). When $\nu(u, o) = 1$, Favre and Jonsson [4] used the valuative tree to characterize the structure of $\mathscr{I}(u)_o$ in dimension two. In [6], Guan and Zhou obtained the following result for arbitrary dimension n, by means of their solution to Demailly's strong openness conjecture [5].

Theorem 1.1. ([6]) *Let u be a plurisubharmonic function on $\Omega \subset \mathbb{C}^n$ with $c_o(u) = 1$. If $\nu(u, o) = 1$, then $\mathscr{I}(u)_o = \mathscr{I}(\log |h|)_o$, where h is the minimal defining function of a germ of regular complex hypersurface through o.*

The *log canonical threshold* (or *complex singularity exponent*) of u at o is defined to be

$$c_o(u) := \sup\{c \geq 0 | \exp(-2cu) \text{ is integrable near } o\}.$$

It is convenient to put $c_o(-\infty) = 0$ (see [3]).

Note that $c_o(u) \cdot \nu(u, o) = 1$ when $n = 1$. In other words, Theorem 1.1 is equivalent to the fact that if there exists a complex line L through o such that $c_{o'}(u|_L) = 1$, then $\mathscr{I}(u)_o = (z_1) \cdot \mathcal{O}_n$ or $\mathcal{O}_n$, in some local coordinates near o.

Thus, it is natural to raise the following question.

Question 1.1. *Let u be a plurisubharmonic function on $\Omega \subset \mathbb{C}^n$ and H an m-dimensional complex plane in $\mathbb{C}^n$ through o. If $c_{o'}(u|_H) = 1$, what is the structure of $\mathscr{I}(u)_o$?*

For the case $m = 1$, Question 1.1 is solved by Theorem 1.1. In the following subsection, we will answer the Question for the case $m = 2$.

1.2. Answer to Question 1.1 for $m = 2$

Firstly, we present the following characterization of multiplier ideals associated with weights of log canonical threshold one in the two-dimensional case.