

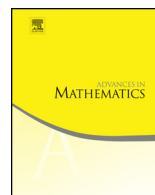


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Weak containment rigidity for distal actions[☆]



Adrian Ioana^{a,*}, Robin Tucker-Drob^b

^a *Mathematics Department, University of California, San Diego, CA 90095-1555, United States*

^b *Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, United States*

ARTICLE INFO

Article history:

Received 29 July 2015

Received in revised form 19 July 2016

Accepted 26 July 2016

Available online 1 August 2016

Communicated by Slawomir J. Solecki

Keywords:

Weak containment

Weak equivalence

Distal action

Compact action

Strong ergodicity

Spectral gap

Factor

Rigidity

ABSTRACT

We prove that if a measure distal action α of a countable group Γ is weakly contained in a strongly ergodic probability measure preserving action β of Γ , then α is a factor of β . In particular, this applies when α is a compact action.

As a consequence, we show that the weak equivalence class of any strongly ergodic action completely remembers the weak isomorphism class of the maximal distal factor arising in the Furstenberg–Zimmer Structure Theorem.

© 2016 Elsevier Inc. All rights reserved.

[☆] A.I. was partially supported by NSF Grant DMS #1161047, NSF Career Grant DMS #1253402, and an Alfred P. Sloan Foundation Fellowship (BR2013-045). R.T.D. was partially supported by NSF Grant DMS #1303921 and by NSF Grant DMS #1600904.

* Corresponding author.

E-mail addresses: aioana@ucsd.edu (A. Ioana), rtuckerd@math.tamu.edu (R. Tucker-Drob).

1. Introduction

The notion of weak containment for group actions was introduced by A. Kechris [16] as an analogue of the notion of weak containment for unitary representations. Let $\Gamma \curvearrowright^\alpha (X, \mu)$ and $\Gamma \curvearrowright^\beta (Y, \nu)$ be two probability measure preserving (p.m.p.) actions of a countable group Γ . Then α is said to be *weakly contained* in β (in symbols, $\alpha \prec \beta$) if for any finite set $S \subset \Gamma$, finite measurable partition $\{A_i\}_{i=1}^n$ of X , and $\varepsilon > 0$, we can find a measurable partition $\{B_i\}_{i=1}^n$ of Y such that for all $\gamma \in S$ and $i, j \in \{1, \dots, n\}$ we have

$$|\mu(\gamma A_i \cap A_j) - \nu(\gamma B_i \cap B_j)| < \varepsilon.$$

If $\alpha \prec \beta$ and $\beta \prec \alpha$, we say that α is *weakly equivalent* to β .

We say that α is a *factor* of β , or that β is an *extension* of α , if there exists a measurable, measure preserving map $\theta : Y \rightarrow X$ such that $\theta(\gamma y) = \gamma\theta(y)$, for all $\gamma \in \Gamma$ and almost every $y \in Y$. The map θ is called a *factor map* or an *extension*. If in addition there is a conull set $Y_0 \subseteq Y$ such that θ is one-to-one on Y_0 , then θ is called an *isomorphism* and we say that α is *isomorphic* to β . The actions α and β are said to be *weakly isomorphic* if each is a factor of the other.

As the terminology suggests, if α is a factor of β , then α is weakly contained in β . The main goal of this note is to establish a rigidity result which provides a general instance when the converse holds.

Theorem 1.1. *Let Γ be a countable group, $\Gamma \curvearrowright^\alpha (X, \mu)$ be a measure distal p.m.p. action, and $\Gamma \curvearrowright^\beta (Y, \nu)$ be a strongly ergodic p.m.p. action.*

If α is weakly contained in β , then α is a factor of β . In particular, if a compact action α is weakly contained in a strongly ergodic action β , then α is a factor of β .

Before recalling the notions involved in [Theorem 1.1](#), let us put it into context and outline its proof.

Weak containment and weak equivalence have received much attention since their introduction. In [16], A. Kechris shows that cost varies monotonically with weak containment and in [17] Kechris uses this monotonicity to obtain a new proof that free groups have fixed price. Several other measurable combinatorial parameters of actions are known to respect weak containment and hence are invariants of weak equivalence; see [1,10,11].

In [3], M. Abért and B. Weiss exhibit a remarkable anti-rigidity phenomenon for weak containment by showing that every free p.m.p. action of Γ weakly contains the Bernoulli action over an atomless base space. The Abért–Weiss Theorem was extended in [21] and used to show that every weak equivalence class contains “unclassifiably many” isomorphism classes of actions, thus ruling out the possibility of weak equivalence superrigidity. These anti-rigidity results stand in marked contrast to the rigidity exhibited in [Theo-](#)

Download English Version:

<https://daneshyari.com/en/article/6425125>

Download Persian Version:

<https://daneshyari.com/article/6425125>

[Daneshyari.com](https://daneshyari.com)