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Degenerate bifurcation of the rotating patches

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A B S T R A C T

In this paper we study the existence of doubly-connected rotating patches for Euler equations when the classical non-degeneracy conditions are not satisfied. We prove the bifurcation of the V-states with two-fold symmetry, however for higher m-fold symmetry with $m \geq 3$ the bifurcation does not occur. This answers to a problem left open in [10]. Note that, contrary to the known results for simply-connected and doubly-connected cases where the bifurcation is pitchfork, we show that the degenerate bifurcation is actually transcritical. These results are in agreement with the numerical observations recently discussed in [10]. The proofs stem from the local structure of the quadratic form associated to the reduced bifurcation equation.

Contents

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1. Introduction

In this paper we deal with the vortex motion for incompressible Euler equations in two-dimensional space. The formulation velocity–vorticity is given by the nonlinear transport equation

$$\begin{cases} \partial_t \omega + v \cdot \nabla \omega = 0, \; x \in \mathbb{R}^2, \; t \geq 0, \\ v = \nabla^\perp \Delta^{-1} \omega, \\ \omega_{|t=0} = \omega_0, \end{cases} \tag{1}$$

where $v = (v^1, v^2)$ denotes the velocity field and $\omega = \partial_1 v^2 - \partial_2 v^1$ its vorticity. The second equation in (1) is the Biot–Savart law which can be written with a singular operator as follows: By identifying the vector $v = (v_1, v_2)$ with the complex function $v_1 + iv_2$, we may write

$$v(t,z) = \frac{i}{2\pi} \int_{\mathbb{C}} \frac{\omega(t,\xi)}{\overline{z} - \overline{\xi}} \, dA(\xi), \quad z \in \mathbb{C}, \tag{2}$$

with dA being the planar Lebesgue measure. Global existence of classical solutions was established a long time ago by Wolibner in [35] and follows from the transport structure of the vorticity equation. For a recent account of the theory we refer the reader to [1,7]. The same result was extended for less regular initial data by Yudovich in [37] who proved that the system (1) admits a unique global solution in the weak sense when the initial vorticity ω_0 is bounded and integrable. This result is of great importance because it enables to deal rigorously with some discontinuous vortices as the vortex patches which are the characteristic function of bounded domains. Therefore, when $\omega_0 = \chi_D$ with D a bounded domain then the solution of (1) preserves this structure for long time and $\omega(t) = \chi_{D_t}$, where $D_t = \psi(t, D)$ is the image of D by the flow. The motion of the patch is governed by the contour dynamics equation which takes the following form: Let $\gamma_t : \mathbb{T} \to \partial D_t$ be the Lagrangian parametrization of the boundary, then

$$\partial_t \gamma_t = -\frac{1}{2\pi} \int_{\partial D_t} \log|\gamma_t - \xi| d\xi.$$