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Multiserial and special multiserial algebras and their representations ☆

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ABSTRACT

In this paper we study multiserial and special multiserial algebras. These algebras are a natural generalization of biserial and special biserial algebras to algebras of wild representation type. We define a module to be multiserial if its radical is the sum of uniserial modules whose pairwise intersection is either 0 or a simple module. We show that all finitely generated modules over a special multiserial algebra are multiserial. In particular, this implies that, in analogy to special biserial algebras being biserial, special multiserial algebras are multiserial. We then show that the class of symmetric special multiserial algebras coincides with the class of Brauer configuration algebras, where the latter are a generalization of Brauer graph algebras. We end by showing that any symmetric algebra with radical cube zero is special multiserial and so, in particular, it is a Brauer configuration algebra.

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1. Introduction

In the study of the representation theory of finite dimensional algebras, the introduction of presentations of algebras by quiver and relations has led to major advances in the field. Such presentations of algebras combined with another combinatorial tool that has proven powerful, the Auslander–Reiten translate and the Auslander–Reiten quiver, have led to many significant advances in the theory, to cite but a selection of these, see for example [14,27,28,41] or for an overview see [7,45,46,8]. The addition of special properties such as semi-simplicity, self-injectivity, Koszulness, finite and tame representation type, and finite global dimension, to name a few, have led to structural results and classification theorems. For example, the Artin–Wedderburn theorems for semi-simple algebras [33], the classification of hereditary algebras of finite representation type [10], Koszul duality [40], classification of Nakayama algebras [8], covering theory of algebras [15], the study of tilted algebras [17,31] and more recently, the study of cluster-tilted algebras beginning with [18,20,6].

Biserial and special biserial algebras have been the object of intense study at the end of the last century. Many aspects of the representation theory of these algebras are well-understood, for example, to cite but a few of the earlier results, the structure of the indecomposable representations [42,29,50], almost split sequences [19], maps between indecomposable representations [22,37], and the structure of the Auslander–Reiten quiver [24]. Recently there has been renewed interested in this class of algebras. On the one hand this interest stems from its connecting with cluster theory. In [5] the authors show that the Jacobian algebras of surface cluster algebras are gentle algebras, and hence a subclass of special biserial algebras. This class has been extensively studied since, see [21,35] for examples of the most recent results. On the other hand with the introduction of τ-tilting and silting theory [2,3], there has been a renewed interested in special biserial algebras and symmetric special biserial algebras, in particular, see [1,38,51,52].

For self-injective algebras, Brauer tree and Brauer graph algebras have been useful in the classification of group algebras and blocks of group algebras of finite and tame representation type [13,16,32,23] and the derived equivalence classification of self-injective algebras of tame representation type, see for example in [4,47] and the references within. In these classifications biserial and special biserial algebras have played an important role.

In this paper, we study two classes of algebras, *multiserial* and *special multiserial* algebras introduced in [49], that are mostly of wild representation type. These algebras generalize biserial and special biserial algebras. In fact, they contain the classes of biserial and special biserial algebras and we will see that they also contain the class of symmetric algebras with radical cube zero. One common feature of these classes is that their representation theory is largely controlled by the uniserial modules. The same is true for multiserial and special multiserial algebras.

We say that a module M over some algebra is *multiserial* if the radical of M is a sum of uniserial modules $U_1, \dots U_l$ such that, if $i \neq j$, then $U_i \cap U_j$ is either (0) or a