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The $\mathfrak{sl}_n$ foam 2-category: A combinatorial formulation of Khovanov–Rozansky homology via categorical skew Howe duality

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ABSTRACT

We give an elementary construction of colored $\mathfrak{sl}_n$ link homology. The invariant takes values in a 2-category where 2-morphisms are given by foams, singular cobordisms between $\mathfrak{sl}_n$ webs; applying a (TQFT-like) representable functor recovers (colored) Khovanov–Rozansky homology. Novel features of the theory include the introduction of "enhanced" foam facets which fix sign issues associated with the original matrix factorization formulation and the use of skew Howe duality to show that (enhanced) closed foams can be evaluated in a completely combinatorial manner. The latter answers a question posed in [42].

Contents

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1. Introduction

The usefulness of Khovanov homology [22] stems not only from its power as a topological invariant, but also from its relatively simple description. Indeed, one can understand Khovanov homology as assigning to a link in S^3 a (co)chain complex in which the "chain groups" are collections of disjoint circles in the plane and whose morphisms are given by saddle cobordisms. A simple operation assigning vector spaces to circles and linear maps to cobordisms (a TQFT) then gives a complex of vector spaces and the homology of this complex is the link invariant. Despite the simplicity of this description, the link invariant is quite powerful; for example, Rasmussen uses Khovanov homology to give an elegant proof of the Milnor conjecture [49], Kronheimer and Mrowka show that it detects the unknot [32], and Grigsby and Ni show that (a variant of) Khovanov homology distinguishes braids from other tangles [17].

Following his construction of $\mathfrak{sl}_2$ link homology, Khovanov introduced a homology theory in the spirit of the invariant mentioned above categorifying the $\mathfrak{sl}_3$ link polynomial [24]. This invariant assigns to a link a complex of trivalent graphs, called webs, with morphisms given by foams, singular cobordisms between such graphs. Again, one can pass from this complex to one consisting of vector spaces and linear maps and compute homology to obtain the link invariant. In subsequent years, various authors introduced link homology theories categorifying the $\mathfrak{sl}_n$ link polynomial for $n \geq 4$. Khovanov and Rozansky gave the initial construction based on the theory of matrix factorizations [31]. In later work, Cautis and Kamnitzer [5,6] construct $\mathfrak{sl}_n$ link homology using derived categories of coherent sheaves on orbits in the affine Grassmannian, Mazorchuk and Stroppel [46] and Sussan [52] give constructions based on category $\mathcal{O}$, and Webster [54]