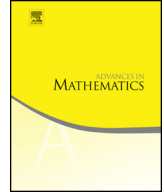




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On the birationality of complete intersections associated to nef-partitions



Zhan Li*

*Rutgers University, Department of Mathematics, 110 Frelinghuysen Rd.,
Piscataway, NJ, 08854, USA*

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ABSTRACT

We prove that generic complete intersections associated to multiple mirror nef-partitions are all birational. This result solves a conjecture by Batyrev and Nill in [6] under some mild assumptions.

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* Current address: Beijing International Center for Mathematical Research, Peking University, Beijing 100871, China.

E-mail addresses: lizhan@math.rutgers.edu, lizhan@math.pku.edu.cn.

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1. Introduction

Mirror symmetry was first discovered in string theory as a duality between families of 3-dimensional Calabi–Yau manifolds. Since its discovery more than twenty years ago, it has drawn much attention from physicists and mathematicians. Among the methods of constructions of mirror pairs, Batyrev and Borisov used the complete intersections in toric varieties [2,8]; Berglund, Hübsch and Krawitz used the finite quotients of hypersurfaces in weighted projective space [7,25]; Gross and Siebert used the toric degeneration of Calabi–Yau varieties to connect the Strominger–Yau–Zaslow approach and the Batyrev–Borisov approach [17,18].

The Batyrev–Borisov construction is one of the best understood settings in mirror symmetry. Batyrev [2] used Δ -regular hypersurfaces in toric varieties associated to reflexive polytopes as a way to construct a large set of mirror pairs. In this case, the mirror pair consists of the family of Δ -regular hypersurfaces associated to a reflexive polytope and the family of Δ -regular hypersurfaces associated to its dual polytope. Borisov [8] generalized Batyrev’s construction by considering nef-partitions of reflexive polytopes. A nef-partition of a reflexive polytope corresponds to a decomposition of the boundary divisor into nef Cartier divisors. In this case, the mirror pairs are constructed as the family of complete intersections associated to a nef-partition and the family of complete intersections associated to its dual nef-partition. These complete intersections are Calabi–Yau varieties, and their string-theoretic Hodge numbers behave as predicted by mirror symmetry [4].

Compared to hypersurfaces, complete intersections associated to nef-partitions are more complicated. In particular, they may exhibit nontrivial multiple mirror phenomenon, i.e. two Calabi–Yau varieties X, \tilde{X} may have the same mirror Y [14] depending on a choice of nef-partition. If this is the case, the homological mirror symmetry conjecture [24] implies that the derived categories of coherent sheaves on X, \tilde{X} are equivalent. Indeed, according to the conjecture, the derived categories of X, \tilde{X} are expected to be equivalent to the Fukaya categories of their mirrors, which in this case are the same because X, \tilde{X} are multiple mirrors.

Besides derived equivalence, Batyrev and Nill asked whether toric multiple mirrors (of any dimension) in the setting of the Batyrev–Borisov construction are birational ([6] Question 5.2). We give an affirmative answer to this question in [Theorem 4.10](#) under some mild assumptions:

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