

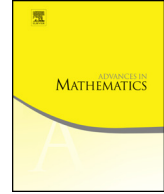


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Flow by powers of the Gauss curvature[☆]



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ABSTRACT

We prove that convex hypersurfaces in \mathbb{R}^{n+1} contracting under the flow by any power $\alpha > \frac{1}{n+2}$ of the Gauss curvature converge (after rescaling to fixed volume) to a limit which is a smooth, uniformly convex self-similar contracting solution of the flow. Under additional central symmetry of the initial body we prove that the limit is the round sphere for $\alpha \geq 1$.

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1. Introduction

In this paper we study the flow of convex hypersurfaces $\tilde{X}(\cdot, \tau) : M \rightarrow \mathbb{R}^{n+1}$ by the α -power of Gauss curvature:

$$\frac{\partial}{\partial \tau} \tilde{X}(x, \tau) = -\tilde{K}^\alpha(x, \tau) \nu(x, \tau). \tag{1.1}$$

Here $\nu(x, \tau)$ is the unit exterior normal at $\tilde{X}(x, \tau)$ of $\tilde{M}_\tau = \tilde{X}(M, \tau)$, and $\tilde{K}(x, \tau)$ is the Gauss curvature of \tilde{M}_τ at $\tilde{X}(x, \tau)$ (the tildes distinguish these from the normalized counterparts introduced below).

Equation (1.1) is a parabolic fully nonlinear equation of Monge–Ampère type, hence the study sheds light on the general theory of such equations. The case $\alpha = 1$ was proposed by Firey [19] as a model for the wearing of tumbling stones. The equation with general powers also arises in the study of affine geometry and of image analysis [1,15,31,33,34]. For large α the equation becomes more degenerate and for small α it becomes more singular. Studying them together gives an example of nonlinear parabolic equations with varying degeneracy. The interested reader may consult [7] for motivation for the study of this flow. For the short time existence, it was proved in [39] for $\alpha = 1$, and for any $\alpha > 0$ in [17] that the flow shrinks any smooth, uniformly convex body $M_0 = \partial\Omega_0$ to a point z_∞ in finite time $T > 0$. An important differential Harnack estimate (also referred as Li–Yau–Hamilton type estimate) was later proved in [18] (see also [2]). The current paper concerns the asymptotics of the solutions as the time approaches to the singular time T .

The study of the asymptotic behavior is equivalent to the large time behavior of the normalized flow, which is obtained by re-scaling about the final point to keep the enclosed volume fixed, and suitably re-parameterizing the time variable (see section 3 for details):

$$\frac{\partial}{\partial t} X(x, t) = -\frac{K^\alpha(x, t)}{\int_{\mathbb{S}^n} K^{\alpha-1}} \nu(x, t) + X(x, t). \tag{1.2}$$

Here we write $\int_{\mathbb{S}^n} f(x) d\theta(x) = \frac{1}{\omega_n} \int_{\mathbb{S}^n} f(x) d\theta(x)$ for any continuous function f on \mathbb{S}^n , where $d\theta(x)$ is the spherical Lebesgue measure and $\omega_n = |\mathbb{S}^n|$, and we interpret K as a function on \mathbb{S}^n via the Gauss map diffeomorphism $\nu : M_t \rightarrow \mathbb{S}^n$. It can be easily checked that $M_t = X(M, t)$ encloses a convex body Ω_t whose volume $|\Omega_t|$ changes according to the equation:

$$\begin{aligned} \frac{d}{dt} |\Omega_t| &= -\frac{1}{\int_{\mathbb{S}^n} K^{\alpha-1}} \int_{M_t} K^\alpha + \int_{M_t} \langle X, \nu \rangle \\ &= -\omega_n + (n + 1)|\Omega_t|. \end{aligned}$$

Hence if $|\Omega_0| = |B(1)| = \frac{\omega_n}{n+1}$, where $B(1) \subset \mathbb{R}^{n+1}$ is the unit ball, then $|\Omega_t| = |B(1)|$ for all t .

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