

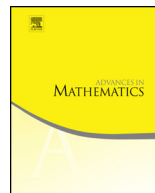


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Weak amenability of Fourier algebras and local synthesis of the anti-diagonal



Hun Hee Lee ^{a,*}, Jean Ludwig ^{b,2}, Ebrahim Samei ^{c,3},
Nico Spronk ^{d,4}

^a Department of Mathematical Sciences and Research Institute of Mathematics, Seoul National University, Gwanak-ro 1, Gwanak-gu, Seoul 08826, Republic of Korea

^b Institut Élie Cartan de Lorraine, Université de Lorraine – Metz, Bâtiment A, Ile du Saulcy, F-57045 Metz, France

^c Department of Mathematics and Statistics, University of Saskatchewan, Room 142 McLean Hall, 106 Wiggins Road, Saskatoon, SK, S7N 5E6, Canada

^d Department of Pure Mathematics, University of Waterloo, Waterloo, ON, N2L 3G1, Canada

ARTICLE INFO

Article history:

Received 16 February 2015

Received in revised form 12 January 2016

2016

Accepted 17 January 2016

Available online 30 January 2016

Communicated by Dan Voiculescu

MSC:

primary 43A30

secondary 43A45, 43A80, 22E15,

22D35, 46H25, 46J40

Keywords:

Fourier algebra

Weak amenability

Local synthesis

ABSTRACT

We show that for a connected Lie group G , its Fourier algebra $A(G)$ is weakly amenable only if G is abelian. Our main new idea is to show that weak amenability of $A(G)$ implies that the anti-diagonal, $\hat{\Delta}_G = \{(g, g^{-1}) : g \in G\}$, is a set of local synthesis for $A(G \times G)$. We then show that this cannot happen if G is non-abelian. We conclude for a locally compact group G , that $A(G)$ can be weakly amenable only if it contains no closed connected non-abelian Lie subgroups. In particular, for a Lie group G , $A(G)$ is weakly amenable if and only if its connected component of the identity G_e is abelian.

© 2016 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: hunheelee@snu.ac.kr (H.H. Lee), ludwig@univ-metz.fr (J. Ludwig), samei@math.usask.ca (E. Samei), nspronk@uwaterloo.ca (N. Spronk).

0.1. Background

Questions on the nature of bounded derivations on (commutative) Banach algebras \mathcal{A} have been around for a long time, in particular vanishing of bounded Hochschild cohomologies $H^1(\mathcal{A}, \mathcal{M})$ for certain Banach \mathcal{A} -modules \mathcal{M} . See, for example, [38,19]. Johnson systematized many of these questions in [16]. In particular, he showed that for a locally compact group G , its group algebra is *amenable* (i.e. $H^1(L^1(G), \mathcal{M}^*) = \{0\}$ for each dual module \mathcal{M}^*) if and only if G is an amenable group. He also started the problem of determining when $H^1(L^1(G), L^1(G)^*) = \{0\}$.

For a commutative Banach algebra \mathcal{A} , Bade, Curtis and Dales [2] introduced the concept of *weak amenability*, which is defined as having $H^1(\mathcal{A}, \mathcal{M}) = \{0\}$ for all symmetric Banach modules. They observed that this is equivalent to having $H^1(\mathcal{A}, \mathcal{A}^*) = \{0\}$. There is an interesting universal module also exhibited by Runde [35]. The above observation of [2], leads us to refer to any Banach algebra \mathcal{B} as *weakly amenable* if $H^1(\mathcal{B}, \mathcal{B}^*) = \{0\}$. Weak amenability was established for all $L^1(G)$ by Johnson [17].

The Fourier algebras, $A(G)$, as defined by Eymard [6], are dual objects to the group algebras $L^1(G)$ in a sense which generalizes Pontryagin duality. It was long expected that the amenability properties enjoyed by group algebras would also extend to Fourier algebras. Hence it was a surprise when Johnson [18] showed that $A(G)$ fails to be weakly amenable for any compact simple Lie group, in particular for $G = \text{SO}(3)$. This motivated Ruan [34] to consider the operator space structure $A(G)$ inherits by virtue of being the predual of a von Neuman algebra. He proved that $A(G)$ is operator amenable if and only if G is amenable. Operator weak amenability for general $A(G)$ was determined by Spronk [39] and, independently, by Samei [36]. The question of amenability of $A(G)$ was settled by Forrest and Runde [8]: it happens exactly when G is virtually abelian. They also showed that if the connected component G_e is abelian, then $A(G)$ is weakly amenable. The following is suggested.

Question 0.1. *If $A(G)$ is weakly amenable, then must G_e be abelian?*

Much progress has been made in answering this question. Building on work of Plymen [32] – which was written to answer a question in [18] – Forrest, Samei and Spronk [9] showed that $A(G)$ is not weakly amenable whenever G contains a non-abelian connected compact subgroup. Exciting recent progress was made by Choi and Ghandehari. In [3] they show for the affine motion group, and hence any simply connected semisimple Lie group, and also for the reduced Heisenberg group, that the Fourier algebra is not

¹ The author was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF), grant NRF-2015R1A2A2A01006882.

² The author was supported by Institut Élie Cartan de Lorraine.

³ The author was supported by NSERC Grant 366066-2014, and the Professeur Invité program at Institut Élie Cartan de Lorraine.

⁴ The author was supported by NSERC Grant 312515-2010, and the Korean Brain Pool Program.

Download English Version:

<https://daneshyari.com/en/article/6425264>

Download Persian Version:

<https://daneshyari.com/article/6425264>

[Daneshyari.com](https://daneshyari.com)