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Motivic weightless complex and the relative Artin motive

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ABSTRACT

We construct a motivic analogue of the weightless cohomology groups [18] and demonstrate several expected properties – ring structure, Kunneth, functoriality for certain morphisms etc. We also demonstrate the expected relationship with the motivic intersection complex of Wildeshaus [26,25] (whenever it exists). The methods also give another construction of the weightless cohomology groups over any base field. In characteristic 0, this motive coincides with the motive $\mathbb{E}_X$ of Ayoub–Zucker [3] and recovers the motivic construction of the reductive Borel–Serre compactification of Shimura varieties.

1. Introduction

1.1. In [9] Goresky–Harder–MacPherson introduced certain complexes of sheaves on the reductive Borel–Serre compactification of a locally symmetric space, which in turn were used to define the weighted cohomology groups associated to an arithmetic group. These groups include the ordinary homology and cohomology of reductive Borel–Serre compactification. If the locally symmetric space is Hermitian (the case of Shimura varieties) they also include the intersection cohomology of its Baily–Borel compactification.

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In [15] S. Morel constructs analogues of weighted cohomology groups over a variety defined over a finite field, by constructing analogous complexes in the derived category of l-adic sheaves. Her construction is also valid in the derived category of mixed Hodge modules and in [17] A. Nair demonstrates that it indeed recovers the weighted cohomology groups in the context of Shimura varieties. One advantage of this construction is that the otherwise *topological* construction of weighted cohomology groups now carry structures expected of the cohomology of an *algebraic* variety (e.g. Hodge filtration, weight filtration).

In [18] we investigate analogous constructions for an arbitrary algebraic variety over either $\mathbb{C}$, a number field, or a finite field and call them weight truncated cohomology groups. Like weighted cohomology groups, these depend on an auxiliary choice of a weight profile. For one choice of "weight profile" they reproduce the intersection cohomology groups and for another choice, they produce what we called the weightless cohomology groups. For Baily–Borel compactification of a Shimura variety, these correspond to ordinary cohomology groups of the reductive Borel–Serre compactification.

The purpose of this article is to construct motivic analogues of weightless cohomology groups for an arbitrary variety defined over any fixed field k. In characteristic 0, we recover the motive $\mathbb{E}_X$ of Ayoub–Zucker [3] – in particular we also construct the motive of the reductive Borel–Serre compactification in the context of a Shimura variety.

1.2. In the conjectural picture of Beilinson one should have an abelian category of mixed motivic sheaves over any scheme X, such that the Grothendieck's six operations are appropriately defined. By work of F. Morel and Voevodsky [14], Jardine [12], Ayoub [1,2], Cisinski–Deglise [7] and others, one has a tensor triangulated category which acts as the derived category of motivic sheaves equipped with the Grothendieck's six functors (we work with $\mathbb{Q}$ coefficients throughout).

Fix any field k and let $DM(X)$ denote the tensor triangulated category of motivic sheaves over a variety X/k (for us $DM(-)$ is characterized by the properties 2.2.2 which are known to hold for the above constructions). Let $\mathbf{1}_X$ denote the unit object in $DM(X)$. Then our main construction is the following:

Theorem *(Theorem 3.3.10). For a variety X over any field k, there are objects $EM_X \in DM(X)$ such that:*

i. If $\pi : \hat{X} \to X$ is the normalization then $\pi_* EM_{\hat{X}} = EM_X$. If X is smooth then $\mathbf{1}_X = EM_X$.

ii. There is a morphism $EM_X \otimes EM_X \to EM_X$ making EM_X into a ring object. There is a natural map of ring objects:

$$\mathbf{1}_X \to EM_X$$

iii. EM_X has pullbacks for any morphism $f : Y \to X$ such that the closed image of each irreducible component of Y meets the regular part of X. That is, in this case there