Advances in Mathematics 292 (2016) 478–528

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Differentiability of thermodynamical quantities in non-uniformly expanding dynamics

T. Bomfim, A. Castro, P. Varandas

Departamento de Matemática, Universidade Federal da Bahia, Av. Ademar de Barros s/n, 40170-110 Salvador, Brazil

A R T I C L E I N F O

Article history:
Received 23 May 2014
Received in revised form 22 October 2015
Accepted 7 January 2016
Communicated by Vadim Kaloshin

MSC:
37A35
37C30
37C40
37D25
60F

Keywords:
Non-uniform hyperbolicity
Regularity of topological pressure
Linear response formula
Regularity of limit theorems

A B S T R A C T

In this paper we study the ergodic theory of a robust non-uniformly expanding maps where no Markov assumption is required. We prove that the topological pressure is differentiable as a function of the dynamics and analytic with respect to the potential. Moreover we not only prove the continuity of the equilibrium states and their metric entropy as well as the differentiability of the maximal entropy measure and extremal Lyapunov exponents with respect to the dynamics. We also prove a local large deviations principle and central limit theorem and show that the rate function, mean and variance vary continuously with respect to observables, potentials and dynamics. Finally, we show that the correlation function associated to the maximal entropy measure is differentiable with respect to the dynamics and it is C^1-convergent to zero. In addition, precise formulas for the derivatives of thermodynamical quantities are given.

E-mail addresses: tbnunes@ufba.br (T. Bomfim), armando@impa.br (A. Castro), paulo.varandas@ufba.br (P. Varandas).

URLs: https://sites.google.com/site/homepageofthiagobomfim/ (T. Bomfim), http://www.pgmat.ufba.br/varandas (P. Varandas).

http://dx.doi.org/10.1016/j.aim.2016.01.017

1. Introduction

The thermodynamical formalism was brought from statistical mechanics to dynamical systems by the pioneering works of Sinai, Ruelle and Bowen [45,11,12] in the mid seventies. Indeed, the correspondence between one-dimensional lattices and uniformly hyperbolic maps, via Markov partitions, allowed to translate and introduce several notions of Gibbs measures and equilibrium states in the realm of dynamical systems. Nevertheless, although uniformly hyperbolic dynamics arise in physical systems (see e.g. [28]) they do not include some relevant classes of systems including the Manneville–Pomeau transformation (phenomena of intermittency), Hénon maps and billiards with convex scatterers. We note that all the previous systems present some non-uniformly hyperbolic behavior and its relevant measure exhibits a weak Gibbs property. More recently there have been established many evidences that non-uniformly hyperbolic dynamical systems admit countable and generating Markov partitions. This is now parallel to the development of a thermodynamical formalism of gases with infinitely many states. We refer the reader to [43,37,36] for some recent progress in this direction.

A cornerstone of the theory that has driven the recent attention of many authors both in the physics and mathematics literature concerns the differentiability of thermodynamical quantities as the topological pressure, SRB measures or equilibrium states with respect to the underlying dynamical system. Results on the differentiability of the topological entropy and pressure include some important contributions by Walters [48] and Katok, Knieper, Pollicott and Weiss [30] on the differentiability of the topological entropy for Anosov and geodesic flows. The differentiability of the SRB measure or equilibrium state with respect to the dynamical system has been referred, for natural reasons, as linear response formulas (see e.g. [41]). This has proved to be a hard subject not yet completely understood. In fact, this has been studied mostly for uniformly hyperbolic diffeomorphisms and flows in [30,39,13,29], for the SRB measure of some partially hyperbolic diffeomorphisms in [22] and for one-dimensional piecewise expanding and quadratic maps in [40,6–8,3] and a general picture is still far from complete. In this paper we address these questions for robust classes of non-uniformly expanding maps.

If, on the one hand, the study of finer statistical properties of thermodynamical quantities as equilibrium states, mixing properties, large deviation and limit theorems, stability under deterministic perturbations or regularity of the topological pressure is usually associated to good spectral properties of the Ruelle–Perron–Frobenius operator, on the other hand neither the stability of the equilibrium states or differentiability results for thermodynamical quantities could follow directly from the spectral gap property. This is due to the fact that transfer operators acting on the space of Hölder continuous potentials may not even vary continuously with the dynamical system (see e.g. Subsection 6.1.1 for an example), which makes the classical perturbation theory hard to apply. Revealing its fundamental importance, the functional analytic approach to thermodynamical formalism has gained special interest in the last few years and produced new and interesting